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Redesigning an Experiment to Determine the Coefficient of Friction

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Summary

Typical high school physics experiments that investigate friction coefficients usually use a weighted mass that is dragged across a surface and is attached to a spring scale; the spring scale measures the frictional force occurring in the system. In such an experiment, a constant velocity (zero net force) is necessary in order to ensure that the measured force is only the friction force. While this configuration is simple to conduct and construct, it can be rather difficult to maintain a constant velocity of the weighted mass and to read the moving spring scale at the same time, which may result in large errors. We attempted to solve these issues by designing a new friction-coefficient experiment involving only static measurements. To conduct our experiment, we slid a block down a sloped surface and observed where the block landed. By measuring the horizontal distance of the block's landing location, we could definitively derive the kinetic friction coefficient between the wood block and the sloped board. The standard deviation from our newly designed experiment is ~3 times smaller than that of the traditional experiment, which demonstrates that our experiment may be a viable replacement for standard high school physics experiments on kinetic friction. This experiment can also enhance the learning of physics topics like Newton's second law, motion in two dimensions, friction, and programming.

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Introduction

In high school physics experiments, the most common way to determine the kinetic friction coefficient is to measure the normal force (F_n) produced by an object placed on a horizontal surface (1). **Figure 1a** displays how such an experiment would be done. F_n can be obtained by taking the mass of the object, then attaching a spring scale and pulling it horizontally until it overcomes the static friction. When the object is moving at a constant velocity, the horizontal net force is zero according to Newton's second law. Thus, the observer can obtain the frictional force (F_μ) by reading the spring scale. Then, one can directly calculate the coefficient of kinetic friction for the setup (*i.e.*, $\mu = F_\mu / F_n$). The measurements of the coefficients of friction from the traditional dynamic experiment are shown in **Figure 1b** for twenty trials using a smooth wood-on-wood surface. The measured data fluctuated significantly. This is because the observer must measure the friction force by reading the moving spring scale while pulling the object at a constant velocity. This makes precise measurement of μ difficult. In order to build a precise system to determine the coefficient of kinetic friction, we redesigned the experiment. By removing the need to dynamically measure any forces, the redesigned experiment avoids the difficulty of keeping a constant velocity in the traditional experiment, which can lead to large errors. The only measurements involved in our newly designed experiment are the length (L) of a sloped board, the height (H_1) of the sloped board, the height (H_2) of the table that the sloped board rests on, and the horizontal distance D, as shown in **Figure 2**. Since all these quantities can be easily measured, we hypothesized that this type of experiment should be more precise in comparison to the traditional experiment. In addition, because this experiment uses well known equations (like Newton's second law, distance formula, motion in two dimensions), performing this experiment can aid in the learning of physics concepts and mathematical skills.

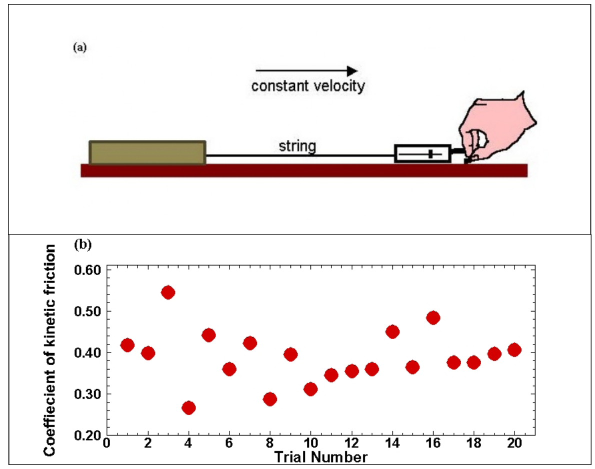

Figure 1: (a) shows the old, traditional experiment set up in high school labs and (b) shows the coefficients of kinetic friction measured using this method.

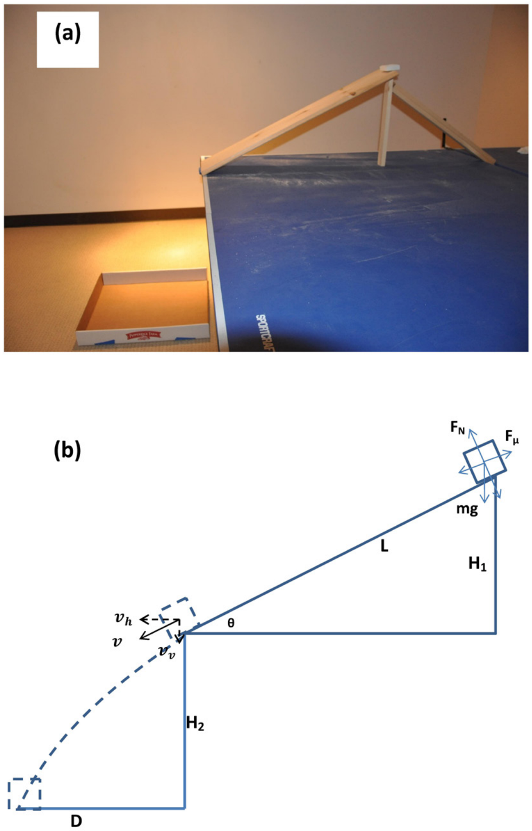

Figure 2: (a) and (b) show our experiment set up. As shown in both figures, a block was initially balanced at the top on its center of mass. It then slid down the sloped surface. Once it reached the end, it fell off with a horizontal and vertical velocity. It impacted the sand-box at distance D.

Results

The results from our new experiment are μ = 0.37±0.023, 0.49±0.017, and 0.50±0.027 for smooth, rough, and sanded surfaces, respectively. The data are presented in **Figure 3**, in which the measurement of the friction between smooth surfaces using the traditional dynamic method is also presented. Interestingly, the average kinetic friction coefficient for the smooth surface was μ = 0.37±0.023, which is reasonably similar, but not exactly the same as the kinetic friction coefficient (μ = 0.30) for wood on wood given in the University of the State of New York's Reference Tables for Physical Setting/PHYSICS, 2006 Edition (2). This discrepancy most likely lies in using different types of wood. The results from the traditional method were μ = 0.388±0.063 for the smooth wood-on-wood surface, and the standard deviation (±16.24%) of the traditional experiment is approximately ~3 times larger than what we got in our new experiment (error-bar of ±6.2%), which reflects the fact that the 20 trials yielded a larger distribution of values for the predicted coefficient. The standard error for the traditional experiment is 0.014, which is larger than the standard error of 0.005 in our new experiment. This confirms that the results we obtained are more precise, producing a much smaller standard deviation than the traditional method.

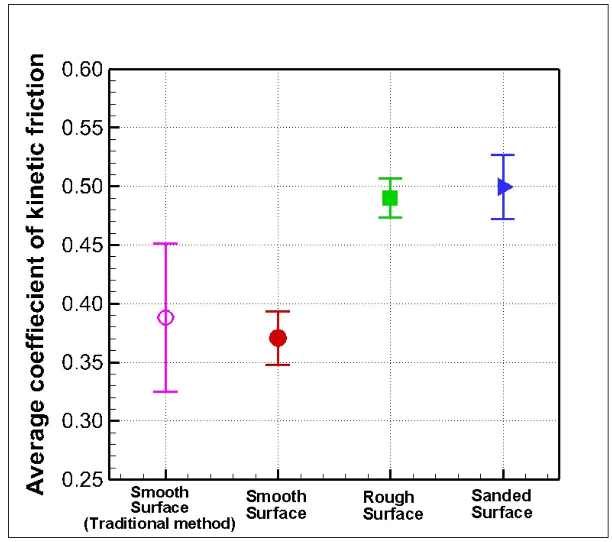

Figure 3: The average kinetic friction coefficient for each surface with error bars marking the standard deviation.

Discussion

Figure 4 shows the expected result that the smooth surface had a lower kinetic friction coefficient, ranging from 0.32 to 0.397, for the twenty trials performed. The rough and the sanded surfaces had similar kinetic friction coefficients at about 0.45 to 0.55, which were higher than that of the smooth surface. The sanded surface had greater variation than the other surfaces. This is likely in part because of the sawdust left on the board. As the block slid down the sloped surface, it could have swept some of the sawdust off the board. Then when the block travels down the same path on a different trial, there is less sawdust left to slow it down. To try to avoid this, we re-sanded the surface every 5 runs. However, this could still be the reason why we see large variation in the measured coefficients for the sanded surface. The other possible source of error may be the fact that the testing block tended to tumble in free-fall. This tumbling made the point of impact harder to determine in the sand box. This could have also caused variation in the data. We expect that the error could be further reduced if a heavier and smaller block were used. We also believe that by adjusting the angle of our sloped board, while keeping the board the same length, we could further improve our experiment. In principle, changing the angle should not change our determined coefficient of friction. This could be done in future experiments to further test and improve our system.

The goal of this project was to measure the coefficient

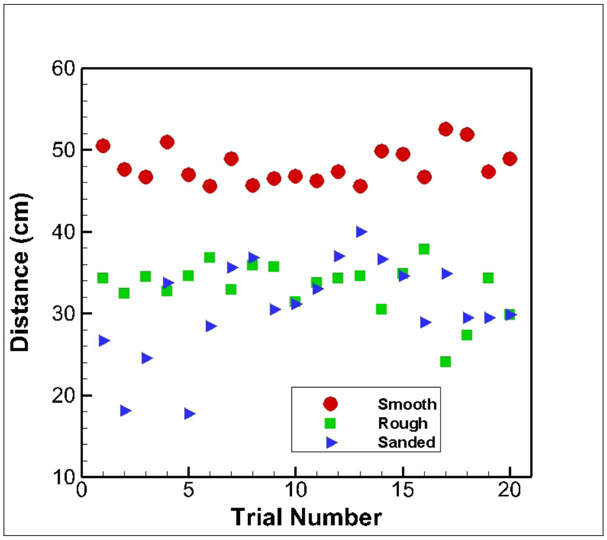

Figure 4: The coefficients of friction for each surface and each trial, calculated with the measured distances listed in Table 1 and the formulae in the theory section.

of friction in a system more precisely. We created a static setup, shown in **Figure 2**, which consisted of a sloped board, a table, and a box with sand to measure the coefficient of kinetic friction. By measuring the length of the sloped board (L) and the heights, H_1 and H_2, as well as D, the horizontal distance that the block falls, we were able to calculate the coefficient of kinetic friction for the wooden surfaces by using a FORTRAN-90 program. The average kinetic friction coefficients and their standard deviations were obtained by averaging the twenty experimental trials for each surface. We compared the smooth surface data to the data measured from the traditional method and found that our new method had a much lower error. In the traditional experiment, the standard deviation was approximately three times greater than the standard deviation in our new experiment. This suggests the traditional experiment is, as we expected, flawed in its precision. Furthermore, our newly designed experiment enhances the learning of physics concepts, mathematical skills, and computer programming, and promotes learning at the high-school level. We hope that this easier and more precise method will be used to teach physics and replace the traditional method. We hope that the derivation of the formulas involved will help high school students hone their physics skills and improve their understanding of mechanics such as acceleration, motion in two dimensions, friction, sloped planes, and free-fall.

After conducting our experiment, we noticed that the coefficients we determined did not quite agree with the result given in the reference book. The wood we used was pine obtained from a local home improvement store, while the type of wood was not specified in the reference book. Since the friction coefficient derived from both the traditional method and the new experiment appears to agree, we conclude that the discrepancy lies in differences between our specific wood samples and those used in the reference book.

Methods

To test whether our newly designed experimental system yielded a more precise measurement of the coefficient of friction, we first established a new experimental setup (**Figure 2a**). The experiment was conducted as follows: a white pine-wood board with a length of L was sloped at a certain angle, θ, set such that the resulting height of the sloped board was H_1. The sloped board was placed at the edge of a table, with the table's height (H_2) measured from the floor box. A wood block with a mass (m), was placed at rest on the top of the sloped board, so that it balanced on its center of mass. The block then slid down the sloped surface and fell off the edge of the table in a parabolic manner. The wood block landed in the sand box on the floor. This was repeated twenty times for each tested surface.

In the experimental setup (**Figure 2**), H_1, H_2, and L were constant for all trials. A trial was defined as one run of the block sliding down the full length of the sloped surface and falling into the box filled with sand. For all experimental trials, the constants were: L = 91.76 cm, H_1 = 45.56 cm, H_2 = 75.57 cm. The tested surfaces were smooth white pine, rough white pine, and sanded white pine with sawdust. The frictional coefficients for these three surfaces should be different, even for the same wood block. The sanded board was re-sanded every 5 trials to ensure the amount of sawdust on the board was approximately the same. The distance traveled by the block, as shown in **Figure 2b**, was measured in every trial by a tape measure. The measurements are recorded in **Table 1** and plotted in **Figure 5**. The distance was greater in the smooth-surface case than in the other two cases. Since there is less friction with the smooth surface, the block had a higher horizontal velocity when it fell off the end of the board and thus traveled farther.

Our setup was built on top of a ping-pong table, as shown in **Figure 2a**. We started by cutting a notch in the vertical support to fit the sloped board. To stabilize the structure, we added a slanted support behind the vertical support (see **Figure 2a**). We then prepared three sloped boards for measuring the coefficient of friction: one with a smooth surface, one with a rough surface, and one with a surface that was sanded with 60-grit sandpaper. Then we acquired a box sufficiently large for our experiment, filled the bottom with sand, and put it directly under the edge of the table (see **Figure 2a**). We measured L, H_1, H_2, and D with a tape measure. The small wooden block was purchased at Home Depot.

# of Trials	Smooth	Rough	Sanded
1	50.5	34.3	26.7
2	47.6	32.4	18.1
3	46.7	34.5	24.5
4	51	32.7	33.7
5	47	34.6	17.78
6	45.6	36.8	28.4
7	48.9	32.9	35.6
8	45.7	35.9	36.8
9	46.5	35.7	30.5
10	46.8	31.4	31.1
11	46.2	33.7	33
12	47.3	34.3	37
13	45.6	34.6	40
14	49.8	30.5	36.6
15	49.5	34.9	34.6
16	46.7	37.8	28.9
17	52.5	24.1	34.9
18	51.9	27.3	29.5
19	47.3	34.3	29.5
20	48.9	29.8	29.8

Table 1: Distances in cm for D measured in the new experiment.

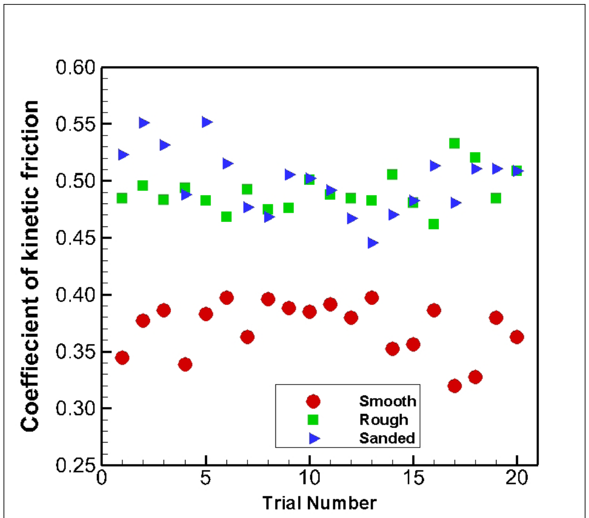

Figure 5: Data collected for the length of D in centimeters. As one can see, the distance traveled for each surface remains relatively concentrated except for the sanded surface.

For each experimental trial, we balanced the block on top of the sloped board and nudged it to initiate movement. The block slid down the slope and fell into the sand box on the floor. We marked the spot where the block first impacted the sand and measured the distance, D, using a measuring tape. The data from these measurements was then inputted into our program that calculated the coefficients of friction. The FORTRAN-90 program also did the statistics, calculating the averaged frictional coefficients and their standard deviations.

Derivation and Theory

The net force acting on the block in this set-up is parallel to the sloped surface and is equal to the gravitational force component minus the friction force (**Figure 2b**). According to Newton's second law, the system of motion along the sloped surface can be described by the following equation:

$$F_{net} = mgsin\theta - mgcos\theta \times \mu = ma\,. \quad (1)$$

In this equation, g is the gravitational constant on earth, which is assumed to be 9.8066 m/s^2 in this experiment; μ is the frictional coefficient between the wood block and the sloped surface, which is the experimentally determined quantity. The variable α is the acceleration of the block when it slides down the sloped surface. From equation (1), we can express the friction coefficient as:

$$\mu = \tan\theta - \left(\frac{a}{gcos\theta}\right). \quad (2)$$

Equation (2) indicates that the friction coefficient is independent of the block mass, and if the acceleration (α) of the block is known, we can determine the friction coefficient. To determine the acceleration, we used the distance and velocity formula:

$$L = v_i t + \frac{1}{2}at^2, \quad (3)$$

$$v = v_i + at. \quad (4)$$

Here, v_t = 0 because the block is at rest initially. By combining the above two equations and eliminating time (t), we obtain $a = v^2/2L$, with the final velocity (v) of the block at the end of the slope. Thus, plugging α into Eq. (2), we can rewrite μ as:

$$\mu = \tan(\theta) - \left(\frac{v^2}{2L \cdot g \cdot cos\theta}\right). \quad (5)$$

Now, if we know the final velocity (v), we can determine μ using the above equation. As the schematic diagram **Figure 2b** shows, the block, having the velocity v at the end of the sloped board, fell off the board at the table edge at a height of H_2. It landed in the sand box at a distance of D, which was measured from the table edge. With distance D, we calculated the falling time, t_2:

$$D = v_h \cdot t_2. \quad (6)$$

This gave the free-fall time:

$$t_2 = \frac{D}{v_h}. \quad (7)$$

Here, v_h is the horizontal component, relative to the ground, of velocity v, as shown by **Figure 2b**. With t_2 in Eq. (7), we can find the vertical component, v_v, since we know the height of H_2, by using the distance formula:

$$H_2 = v_v \cdot t_2 + \frac{1}{2} g t_2^2, \quad (8)$$

We substituted Eq. (7) into the above equation and obtained:

$$H_2 = v_v \cdot (\frac{D}{v_h}) + \frac{1}{2} g (D/v_h)^2. \quad (9)$$

Because v_h in this system is equal to vcosθ and that v_v is equal to vsinθ, we substituted them into Eq. (9) to get:

$$H_2 = vsin\theta \cdot (\frac{D}{vcos\theta}) + \frac{1}{2} g (D/vcos\theta)^2. \quad (10)$$

Because tanθ = sinθ/cosθ, the above Eq. (10) can be simplified as:

$$H_2 = tan\theta \cdot D + \frac{1}{2} g (D/vcos\theta)^2. \quad (11)$$

This yielded the following expression:

$$(\frac{2H_2}{g} - 2tan\theta \cdot \frac{D}{g})/D^2 = 1/(v^2 cos^2\theta), \quad (12)$$

Thus, we further simplified it to isolate v^2:

$$v^2 = \frac{gD^2}{(2H_2 - 2tan\theta \cdot D) \cdot cos^2\theta}. \quad (13)$$

Substituting the expression for v^2 into Eq. (5), we obtained:

$$\mu = \tan(\theta) - \left(\frac{\frac{gD^2}{(2H_2 - 2tan\theta \cdot D) \cdot cos^2\theta}}{2L \cdot g \cdot cos\theta} \right), \quad (14)$$

This can be finally simplified into:

$$\mu = \tan(\theta) - \left(\frac{D^2}{4L \cdot cos^3\theta \cdot (H_2 - D \cdot tan\theta)} \right). \quad (15)$$

Because $cos\theta = \sqrt{L^2 - H_1^2}/L$ and tanθ= $H_1/\sqrt{L^2 - H_1^2}$, we only have to measure the quantities of H_1, H_2, L, and D to determine the coefficient of kinetic friction between the wood block and wood surface using Eq. (15).

The data presented in **Table 1** and **Figure 5** was calculated using Eq. (15) in a FORTRAN-90 program, shown in the appendix. The calculated friction coefficients for the three testing surfaces are displayed in **Figure 4**. The average kinetic friction coefficients ($\bar{\mu}$) and their standard deviation (δμ) were calculated using the following formula:

$$\bar{\mu} = \frac{1}{20} \times \sum_{i=0}^{20} \mu_i \;, \quad (16)$$

$$\delta\mu = \sqrt{\frac{1}{20} \times \sum_{i=0}^{20} (\bar{\mu} - \mu_i)^2} \;, \quad (17)$$

The standard error of the mean is defined as $\sigma_m = \delta\mu/\sqrt{20}$. These calculations have also been coded in the same FORTRAN-90 program that can be available upon request (please email questions@emerginginvestigators.org).

References

1. Wilson, Jerry D., Anthony J. Buffa, and Bo Lou. *Physics*. Seventh ed. San Francisco: Addison-Wesley, 2010.
2. University of the State of New York's Reference Tables for Physical Setting/PHYSICS, 2006 Edition.