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Article

Threshold frequency of a bubble is positively correlated to the density of the surrounding fluid

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SUMMARY

When a fluid is oscillated vertically, under certain parameters, bubbles may move downwards instead of rising. Prior publications defined the threshold frequency as the frequency at which bubbles oscillate in a stationary state. Beyond the threshold frequency, however, bubbles move downwards. This paper investigated the hypothesis that the threshold frequency will increase as the density of the oscillating fluid increases. The parameters in this study were restricted to the frequency of the oscillating fluid and the density of the liquid. As the fluid density increased, the threshold frequency was measured to be higher. Our data analysis concluded that the threshold frequency is positively correlated to the density of the surrounding fluid.

INTRODUCTION

Bubble dynamics have a wide range of potential applications from aerospace to medicine. Although their motions are generally predictable, abnormal effects often occur under vertical oscillations, such as those commonly found in transportation mechanics including fuel tanks of rockets. At frequencies higher than the threshold oscillation frequency, bubbles will sink towards the bottom of the container (1). These sinking bubbles may interfere with the mechanics and likely impose problems (2). To address this issue, we conducted a thorough analysis to determine the existence of a threshold frequency that is integral in deciding the motion of the bubble. We conducted this experiment to test our hypothesis that the increase in fluid density would also increase the threshold frequency of the bubble as the buoyancy force and added mass of the surrounding fluid would decrease (**Figure 1**). The results also indicated that the position of the bubble was a major factor influencing the dynamics of the bubble.

In this analysis, the bubble volume pulsations caused by Laplace pressure are taken into account. The Laplace pressure and the volume pulsations are dependent on the external and internal pressure of the bubble, which are caused by fluid oscillations (3). Since these bubble pulsations are considered to be isothermal, the following condition is applied:

$$P_t V_b = P_0 V_0 \quad \text{(Equation 1)},$$

$$P_t = P_0 + \rho x(g + A\omega^2 \sin\omega t) \quad \text{(Equation 2)},$$

P_0: external pressure (1.013x10^5 Pa), V_0: volume of bubble near surface, V_b: current volume of bubble, P_t: current fluid pressure exerted on bubble, g: gravitational acceleration constant

The following equation can be written as:

$$V_b = \frac{V_0}{1 + \frac{\rho x}{P_0}(g + A\omega^2 \sin\omega t)} \quad \text{(Equation 3)},$$

This indicates to the notion that the volume of the bubble is dependent on its depth, and oscillation.

During the course of a motion through a fluid, an object experiences a force exerted opposite to its relative movement. This is known as drag force and is given as follows (4):

$$F(\dot{x}) = 4\rho R^2 \Psi(Re)\dot{x}^2 \quad \text{(Equation 4)},$$

R: radius of the bubble, $\Psi(Re)$: coefficient of resistance.

Archimedes' Principle states that the upward force on the submerged bubble is identical to the weight of the fluid displaced by the same bubble. This is attributed to the pressure difference in the upper and lower portion of the object. The bottom end will experience higher pressure and will accelerate the object upwards. This phenomenon is expressed as follows:

$$F_B = -\rho V_b g \quad \text{(Equation 5)},$$

ρ: density of fluid, V_b: bubble volume.

When an object moves through the liquid, it uses energy to not only move its own mass but its surrounding fluid as well. This additional work done by the object can be interpreted as added mass. The volume of the added mass can be derived as (5):

$$I = \frac{2}{3}\pi R^3 \quad \text{(Equation 6)},$$

I: volume of added mass, R: radius of the bubble

Calculate mass with ρ, the density of the liquid:

$$m_{add} = \frac{2}{3}\pi\rho R^3 \quad \text{(Equation 7)},$$

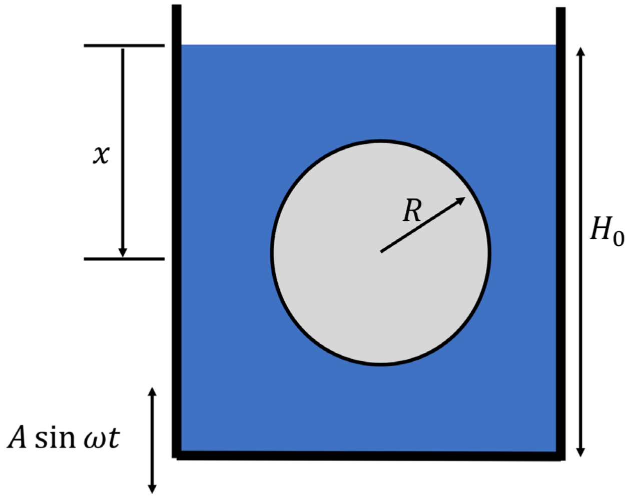

Figure 1. Electron microscopy images of different seperators.

Equation [6] indicates that the mass of the added liquid is half of the displaced liquid. There are additional variations of attached mass that consider distinct conditions including when bubbles are positioned near plane wall or free surface. These other variations, however, can be ignored in our experimental model as the ratio between the depth of bubble *x* and radius of bubble *R* exceeds the value of 8 (6). The *F=ma* form of Newton's Second Law would not be appropriate for this situation as the mass is not constant. Since the added mass changes with the bubble size, a different variation of Newton's Second Law must be employed, in which attached mass variations are taken into account (7).

$$F = (m + m_{add})\ddot{x} + \dot{m}_{add}\dot{x} \qquad \textit{(Equation 8)},$$

Considering all the forces, the following is achieved:

$$(m + m_{add})\ddot{x} + \dot{m}_{add}\dot{x} == -F(\dot{x}) - (\rho V_b - m)(g + A\omega^2 \sin \omega t) \qquad \textit{(Equation 9)}.$$

In summary, equation [9] demonstrates that the "sinking bubble" phenomenon occurs when the added mass-induced forces exceed the buoyancy and drag forces.

RESULTS

In order to determine the threshold frequencies for the three fluids, their motions were first recorded on a 240 fps high definition camera and later computed into graphical representations in video analysis. The motions of 5 bubbles were followed for each frequency. To achieve greater accuracy, we controlled the bubble radius and initial depth of the bubble.

The threshold frequency ω_0 in 99% ethanol (C_2H_5OH) is 30 Hz. The same "sinking bubble" phenomenon observed at

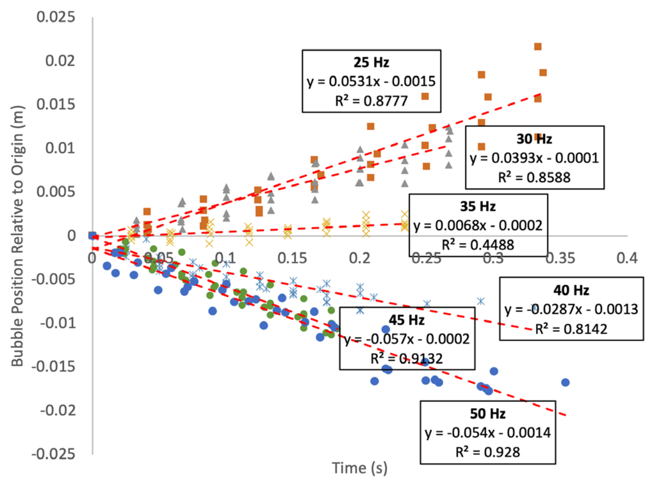

Figure 2. 99% C_2 H_5 OH Bubble Position vs Time from 25 Hz to 35 Hz at an interval of 5 Hz.

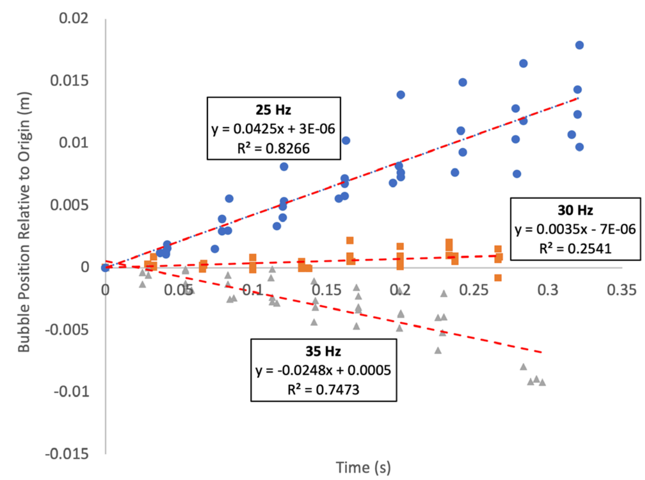

Figure 3. H_2 O Bubble Position vs Time from 25 Hz to 50 Hz at an interval of 5 Hz

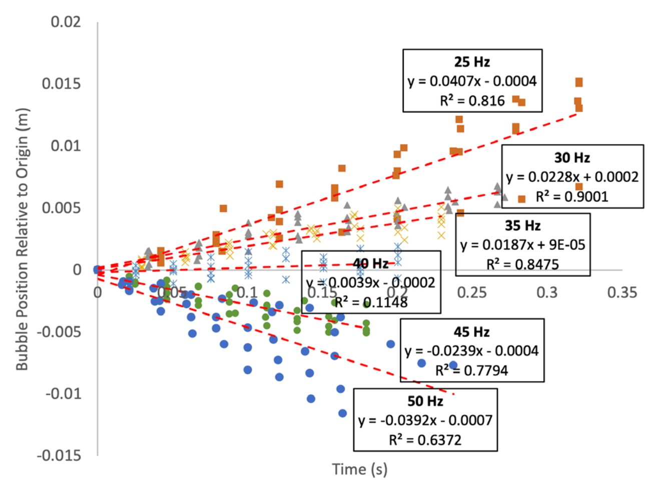

Figure 4. 50% C_3 H_8 O_3 Bubble Position vs Time from 25 Hz to 50 Hz at an interval of 5 Hz

35 Hz was also observed at 40 Hz, 45 Hz, 50 Hz, 55 Hz, and 60 Hz. The only minor difference is the angle of the negative slope, which becomes increasingly negative as the frequency of the oscillation increases from 35 to 60 Hz (**Figure 2**).

In water (H_2O), the threshold frequency ω_0 is 35 Hz (**Figure 3**). The velocity of the "sinking bubble" was observed to be correlated to the frequency of the oscillation. Parallel to our findings in ethanol, as the oscillating frequency increases from 35 Hz to 50 Hz, the slope becomes increasingly negative. Moreover, there is a larger slope differential from 30 Hz to 35 Hz and 35 Hz to 40 Hz compared to other frequencies.

The threshold frequency ω_0 is 40 Hz in a solution of 50% glycerol ($C_3H_8O_3$) and 50% water (H_2O) (**Figure 4**). As the frequency increased beyond 50 Hz, the separation between the two liquids (C_2H_5OH and H_2O) became more prominent. This can be attributed to the distinct density and mass of the two liquids. Furthermore, in frequencies higher than 50 Hz, the small dimensions of bubbles prohibited an accurate analysis of their motions.

DISCUSSION

The threshold frequencies determined in this study are not exact values, but rather experimentally calculated values (Table 1). The threshold frequency was defined as the frequency at which fewer than five bubbles displayed non-stationary motion. Nevertheless, the data indicates that the threshold frequency is correlated with the density of the fluid. As the density of the fluid increased, the threshold frequency increased. The rationale behind this correlation between fluid density and threshold frequency can be attributed to the decreased added mass and buoyancy force.

Equation [4] highlights that the buoyancy force, the force responsible for the rising motion of the bubble, is dependent on the density of the fluid. It can be concluded that a decrease in fluid density will decrease the magnitude of buoyancy force and thus allow the sinking phenomenon to occur at a lower frequency. However, it is valid that Equation [2] suggests otherwise. Equation [2] emphasizes that the density of the fluid influences the volume of the bubble. Considering all other variables are kept constant and only the density of the fluid is increased, the bubble is likely to undergo more distinct changes. In short, increase in fluid density will further decrease the volume of the bubble and decrease the magnitude of buoyancy force. Equation [2] and [4] suggest two contrasting relationships between fluid density and buoyancy force. However, when the values themselves are taken into account, it is evident that the value for fluid density is far larger than the bubble volume. To conclude, decrease in fluid density decreases the magnitude of buoyancy force.

The added mass of the surrounding fluid is dependent on fluid density, as indicated by Equation [6]. The equation ascertains that decrease in fluid density will result in decreased added mass. As the added mass is an integral factor in determining the motion of the bubble, we can indicate that the decrease in added mass will increase the impact of oscillations. The fundamental equation (*F=ma*) indicates that when a constant force is applied the mass of the object will directly determine its acceleration. Applying this notion to the motion of the bubble, when the added mass decreases, the acceleration of the bubble will increase. Increased acceleration results in decreased threshold frequency as there is less force needed to be applied for the bubble to sink. In accordance with our hypothesis, the data indicated that a decrease in fluid density lowers the threshold frequency, as it decreases added mass and increases the acceleration of the bubble.

Fluid Type	Fluid Density	Threshold Frequency ω_0
99% C_2H_5OH	789 kg/m^3	30Hz
H_2O	997 kg/m^3	35Hz
50% $C_3H_8O_3$	1139 kg/m^3	40Hz

Table 1. Threshold Frequencies of Three Fluids with Varying Density

Human error was minimized through our experimental design which considered a minimum of five trials for each frequency. It is true, however, that parameters including fluid density, driving frequency, fluid properties, and bubble size have a significant role in achieving the "sinking bubble" phenomenon. With this in consideration, this experiment explored the correlation between density and driving frequency alone by reducing the significance of other variables. The bubble radius was controlled between certain boundaries to promote consistency in exerted forces and added mass. Moreover, we attempted to control fluid viscosity and other fluid properties of the $C_3H_8O_3$ solution through significant dilutions.

MATERIALS AND METHODS

The following experimental setup was used to generate bubbles at controlled oscillation frequencies for all tests (Figure 5). A 50 ml cylindrical container containing 45 ml fluid (either 99% C_2H_5OH, H_2O, or 50% $C_3H_8O_3$) was placed on the center of the cone of a 30 watt subwoofer run by a DC regulated power supply (Protek PL-3003S). A balance was used to ensure that the container was positioned consistently. A 240 fps camera recorded the data, and a tripod was utilized to ensure that the video footage was not susceptible to any oscillations.

The experimental amplitude was fixed at 0.005 m by an amplifier (Sound Stream U.S.A Handover Hi-Fi Stereo 2 CHANNEL SS-100). The oscillating frequency was varied by a computer application (Tone Generator) at an interval of 5 Hz, ranging from 25 Hz to 50 Hz. For every interval, a 240 fps video was recorded. The durations of the videos were one minute from the start of the oscillation. The container was then replaced with another containing a different fluid. The amplitude was determined through video analysis (Logger Pro 3.15).

The variable H_0 was fixed to the origin point where the motion of the bubble was first recorded. The radius of the bubbles were consistently between 1 mm and 1.2 mm, as larger or smaller bubbles were excluded from the analysis.

This allowed the relevant parameters including buoyancy force, drag force, and added mass to be relatively constant.

Due to the limitations of the video and software, the motion of the bubble could not be continuously analyzed through the tracker application. Therefore, during the data analysis, the lowest point of each cycle was plotted. For instance, when the container oscillates once in 4 frames, the bubble was plotted every four frames when one oscillating period was completed. This allowed a conclusive observation of the general motion of the bubble. The depth of the bubble was restricted to the following parameter: $0.043<x<0.063$.

This depth range was large due to trial to trial variations. For each frequency, five bubbles were analyzed to ensure the accuracy of the general motion. Thus, a total of 90 bubbles were analyzed (6 frequencies each for 3 fluids).

Although bubble dynamics are generally predictable, we identified that abnormal motions occur in certain conditions of oscillation. The threshold frequency marks the lowest frequency at which this anomaly can be observed. We further established that a correlation exists between the threshold frequency and the fluid density. Our examination of such anomaly in bubble dynamics carries seminal significance as motions of bubbles have wide range of applications in fields such as aerospace and medicine.

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