

# Article

# The Effect of Bead Shape and Texture on the Energy Loss Characteristics in a Rotating Capsule.

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#### SUMMARY

Energy loss characteristics in a rotating bead-filled capsule are impacted by bead shape, size, surface texture, density, and capsule fill level. In this paper, the energy loss results are reported from experiments of bead-filled cylindrical capsules rolling down a ramp in a controlled manner without sliding. These experiments were motivated by the potential applications of beadfilled capsules for energy dissipation and vibration control systems, as well as for understanding energy use in rotating drum-based industrial processes. The experiment used a plexiglass cylindrical capsule filled with beads of different sizes, materials, and textures. The bead-filled capsule was rolled down an inclined ramp and came to a stop on a level ramp. The position at which the capsule stopped was recorded. As the capsule rolled down the ramp, the potential energy converted to kinetic energy. When the kinetic energy dissipated, the capsule came to a stop. The distance traveled by the capsule is controlled by the energy loss caused by the collision and the friction of the beads. The nature of the collision and friction of the beads depended on the amount, size, type, and texture of the beads in the cylinder. The data was analyzed to determine the conditions for optimal fill level, which is defined as the case for maximum energy loss. We found that the intermediate filling level had the most energy loss. This optimal filling was obtained for each bead type. The maximum energy dissipation increased with bead surface texture and bead density but decreased with bead size.

# **INTRODUCTION**

Investigating energy loss characteristics in rotating beadfilled capsules is of interest in many fields, including space structures or machine foundations for vibration damping; and chemical, pharmaceutical, and metallurgical industries related to mixing and processing. When the bead-filled capsule rotates, the beads collide with each other and with the wall of the capsule causing energy loss. In addition, energy loss is caused by friction between beads hitting each other and the wall. Therefore, the bead characteristics, such as shape, density, and texture, need to be investigated to understand the energy loss. The focus of this paper was to explore the effect of bead properties upon the energy loss behavior. To investigate the problem, we rolled bead-filled cylindrical capsules down a ramp in a controlled manner without sliding. When the capsule is at rest, it possesses potential energy. Potential energy is defined as U = mgh where U is the total potential energy, m

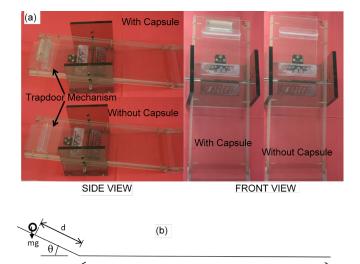
is the mass, g is the acceleration due to gravity, and h is the height at which the capsule rests, measured from a reference plane (1). If the capsule is subjected to rolling, the potential energy will convert into kinetic energy. Based on the potential energy formula, for a given height, since the acceleration due to gravity remains constant, the potential energy at rest will change when the mass of the capsule is changed by adding or removing beads.

Due to the Law of Conservation of Energy, the potential energy will convert into kinetic energy from the moment the capsule begins to roll down the ramp. In addition, energy will be dissipated due to the collision and friction among the beads in the capsule as well as the interaction of the capsule and the ramp. As the energy is dissipated, the capsule will slow down and come to a complete stop. Thus, the energy loss behavior can be studied by measuring the distance travelled by the capsule. Similar experiments have been reported by Balan and Dragomir et al. (2-4) and similar principles are used in a dead blow hammer (5). The effect of rotation speed upon energy loss in rotating cylinders filled with granular materials has been studied by many investigators. Recently, the effect of particle shape on energy dissipaters has been studied using computer simulations (6); however, the role of particle or bead characteristics on energy loss behavior has seldom been reported.

Prior to beginning the experiments, we created several hypotheses. We first hypothesized that since the potential energy increases with mass, and since for larger numbers of beads the energy loss due to collisions will be less due to lack of free space as the fill level of the capsule is increased, the resultant energy loss will be less. Consequently, the distance travelled by the capsule will increase. Secondly, we hypothesized that although the energy loss will be less with the filling-level, the bead texture will nevertheless affect the frictional energy loss. Since the textured beads are expected to have more frictional loss, we expect that if the beads are textured, the fill level at maximum energy loss will decrease. Thirdly, since the shape of the beads affects the inter-bead collisions and friction, we hypothesized that if the bead is non-spherical, then it will dissipate more energy. Finally, since the intensity of energy loss depends upon the bead material density, we hypothesized that the beads with higher density would dissipate more energy.

Our hypothesis that the energy loss decreases with the fill level was not supported by the experimental observations. The hypothesis predicted that the fill level versus position to stop would have an increasing linear trend; however, the





**Figure 1: Experimental apparatus used for measurements. (a)** The photographs give the front and side views of the apparatus with and without the capsule. The trap door mechanism on the inclined ramp and part of the level ramp are visible. (b) Schematic of the capsule at its rest position prior to rolling down the ramp.

data obtained demonstrated a U-shaped behavior, indicating an intermediate optimal fill level where the energy loss was maximized. The remainder of the hypotheses were supported by the experimental results, which showed that the textured, non-spherical, and higher density beads dissipated more energy.

## RESULTS

To conduct the experiment, a ramp arrangement was used (Figure 1a-b). The top of the inclined ramp had a trap door mechanism where a plexiglass cylindrical capsule rested (Figure 1). The capsule was filled with beads of different sizes, materials, and textures (Materials and Methods, Tables 1 and 2). Furthermore, the aspect ratio, given as the ratio of the height to width, for each bead calculated using the bead dimensions is seen to vary from, 0.97 to 2.116 (Table 1 and 2). The larger the aspect ratio of a bead is, the less spherical it is. Multiple fill levels were tested.

A lever was used to lower the trap door mechanism, which sent the capsule into a free roll such that its rolling motion was unimpeded by any obstacle, such as the side walls, and the capsule rolled without sliding. During the free roll, as the potential energy was converted into kinetic energy, the bead filled capsule lost energy due to internal bead-bead and beadwall collision and friction. The energy loss caused the capsule to slow down and come to rest on the flat ramp. When the capsule came to a complete stop, its position was measured using a scale on the side of the flat ramp. A summary of the quantitative and descriptive analysis for each bead-type, including: bead photograph, bead texture index, bead aspect ratio, the smallest "distance to stop", the maximum energy loss and the optimal fill level for "distance to stop" and maximum energy loss are tabulated for convenient comparison (**Table**  1). The measured bead density, bead dimensions expressed as the height and width, and equivalent diameter obtained as the harmonic mean of the height and width are also calculated (**Table 2**).

The standard deviation calculated at each fill level, based upon the number of repeats, has been included on the graphs. Furthermore, the coefficient of variation, defined as the ratio of standard deviation to the average expressed as

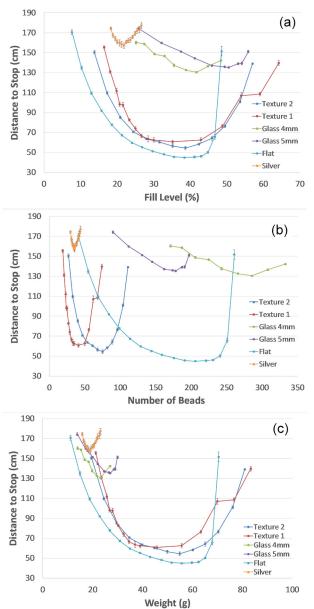


Figure 2: Distance to stop (cm) as a function of fill level, number of beads and weight. (a) shows a general U-shaped trend indicating that each bead has an optimal fill level at which the most energy is dissipated. (b) shows a general U-shaped trend, however, the optimal number of beads at which the most energy loss occurs varied widely amongst the bead types. (c) also shows a general U-shaped trend. In (c) the optimal weights for the metallic beads (Texture 1, Texture 2 and Flat) were comparable to each other and the optimal weights for the non-metallic beads (Glass 4 mm, Glass 5 mm, and Silver) were similar to each other. Each measurement was repeated 10 times.



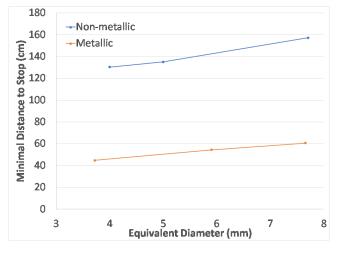


Figure 3: The smallest distance to stop (cm) as a function of the equivalent diameter (mm) for non-metallic and metallic beads. Weight and density normalization is not sufficient to eliminate the data separation for non-metallic and metallic beads suggesting a dependence on other factors governing energy loss. Each measurement was repeated 10 times.

a percent, was found to show the relative variability in the data. The variability in the data was considerably low, which means that the experiment was reproducible. However, some sources of variability could include a difference in the initial placement of the capsule at the starting point, trap door operation, humidity and temperature of the room, and track erosion due to repeated trials.

The average "distance to stop" (including the standard deviation) was plotted against the percent fill level to understand the energy loss behavior (**Figure 2a**). The graphs show a general "U-shaped" trend of the "distance to stop." When the fill level was low, then the energy loss was low, and when the fill level was high, the energy loss was also low. This indicated that there was an optimal fill level at which the most energy was dissipated. With bead filling, the energy loss will only increase. Therefore, the appropriate optimization condition will always be maximum energy loss.

When the capsule was empty, the energy loss was only due to the frictional-type interaction with the ramp. When the capsule was filled in a way that the filled beads movements was extremely restricted, for instance at 100% fill level, the bead collision would be minimal and the tangential rubbing motion between the beads would be limited. When these conditions were met, the energy loss from bead interactions would be minimal and would mainly occur from the frictiontype interaction between the capsule and the ramp. Since the potential energy, kinetic energy, and frictional loss due to the capsule-ramp interaction were all directly proportional to the mass, then for a given capsule that was either empty or 100% full, the mass would not make a difference and the capsule, regardless of its mass, would travel the exact same distance. This was further exemplified in the "Method of Analysis" in the Materials and Methods section from the energy balance

Bead Type	Photo	Texture Index	Aspect Ratio	Smallest Distance to Stop (cm)	Maximum Energy Loss (N.m)		ll level (%) at Maximum Energy Loss
Silver		1	1.081	157.2	0.038	23	23
4mm Glass	0	1.5	1	135.2	0.045	42	42
5mm Glass	0	1.5	1	130.4	0.046	51	53
Texture 1		4	0.97	60.7	0.072	35	49
Texture 2		5	1.166	54.4	0.079	39	50
Flat		2.5	2.116	44.8	0.082	38	45

Table 1: Bead data.

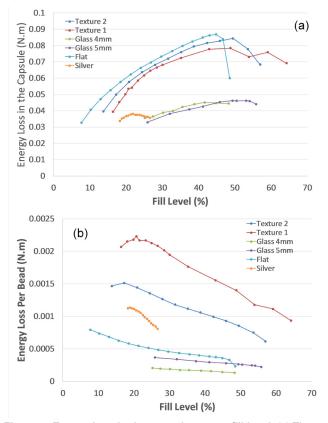
Bead Type	Density (g/cm <sup>3</sup> )	Height (mm)	Width (mm)	Equivalent Diameter
Silver	2.47	7.42	8.02	7.708
Glass 4mm	2.57	4	4	4
Glass 5mm	2.49	5	5	5
Texture 1	6.36	7.77	7.54	7.653
Texture 2	6.36	5.48	6.39	5.9
Flat	6.35	2.74	5.8	3.721

Table 2: Data from aspect ratio and density determination.

given in Eq. 6 for the empty capsule and Eq. 8 for the filled capsule. Since the potential energy and the friction loss due to capsule-ramp interaction for the empty capsule are proportional to mass, the effect of mass cancels out. Moreover, in Eq. 8, when energy loss from bead interactions was negligible, the relation becomes independent of mass. The "U-shaped" trend of the "distance to stop" can be partly explained by the mass independence of energy loss in these extreme conditions. However, for the other cases of fill level, the energy loss within the capsule happens in a complex manner and the mass of the beads in the capsule can affect the energy loss.

The "distance to stop" versus the number of beads also showed a general "U-shaped" trend (**Figure 2b**). However, the optimal number of beads at which the most energy was dissipated is vastly different among the types of beads. Moreover, the "distance to stop" plotted against fill weight. also shows a "U-shaped" trend (**Figure 2c**). Notably, for the case of the percent fill level, the optimal values for the





**Figure 4: Energy loss in the capsule versus fill level.** (a) The energy loss (N.m) in the capsule as a function of fill level (%) showing the maximum energy loss at the optimal fill level. (b) The energy loss per bead as a function of fill level (%) showing a gradual decrease in per bead loss irrespective of the material type. Each measurement was repeated 10 times.

various bead types fell within a narrow range. All the minimal "distance to stop" falls between the 20 and 50% fill levels (**Figure 2a**). Comparing the data, it appears that the minimal "distance to stop" is attained if the fill level is within the range 20-50% irrespective of the number of beads or the filling weight (**Figures 2b and c**). This indicates that at this fill level range, the collision and friction between the beads is optimal for the most energy loss.

On the other hand, if the beads still weighed in the optimal weight range or the fill percent was between 20-50%, but the number of beads was decreased by choosing larger beads, then the energy losses are expected to be reduced. If the bead size was larger, then the number of beads that fit into the capsule decrease. The decrease in energy loss in relation to the bead size can be seen by comparing the smallest "distance to stop" (Figure 3). To make a meaningful comparison, the data for metallic beads and non-metallic beads are plotted separately (Figure 3). From that it can be seen that as the effective bead diameter increases, the smallest "distance to stop" increases. The number of bead interactions are expected to be more between smaller sized beads than larger sized beads. The smaller beads had more energy dissipation because there were more beads in the optimal fill level where the most energy loss occurred. One reason for the separation

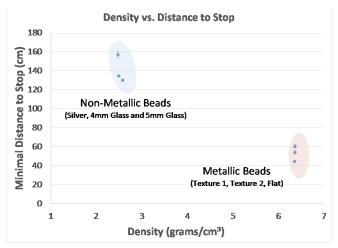


Figure 5: The minimal distance to stop (cm) as a function of bead density (g/cm3). The data shows clustering around similar density beads. The standard deviations of the measurements are also shown; however, they do not show in this scale as they are extremely small. Each measurement was repeated 10 times.

in the plots for metallic and non-metallic beads could be the bead weights. However, the energy loss phenomenon is more complex and is affected by the collision dissipation behavior, as well as the frictional loss due to texture for the two types of bead materials. Therefore, a direct comparison between metallic and non-metallic beads cannot be made. Regardless, it is interesting that both material types follow a similar trend with respect to bead size. Further testing must be done to study the relationship between the size of the beads and the inner diameter of the capsule.

The energy loss in the capsule,  $W_{_{NC}}$ , calculated from Eq. 8, is plotted against the fill level (**Figure 4a**). As expected from the "distance to stop" data, the energy loss is higher in bead types for which the "distance to stop" was smaller. However, the fill level at the maximum energy loss within the capsule was not always in agreement with the fill level at the smallest "distance to stop." The smallest "distance to stop" was determined by the combination of energy loss within the capsule and the capsule-ramp interaction. In contrast, the maximum energy loss within the capsule and the capsule-ramp interactions are complex and depend upon the motion of the beads within the capsule as well as the beads energy dissipation potential, which in turn depends upon the bead's properties, including their size, shape and composition.

The per-bead energy loss shows a gradual decrease in most cases as the fill level increases (**Figure 4b**). The number of beads in the capsule are higher at higher fill levels. This will result in a reduction of the freedom of bead motion. The observed decrease in energy loss per bead with the increasing fill level indicates that the intensity of bead interactions diminishes as the beads lose their freedom of motion. The per bead energy loss also indicates that the energy loss mechanisms are dependent both upon bead texture and bead density. The energy loss per bead for bead types with a high texture index (Texture 1 and Texture 2) was



much larger than that for low texture index (Flat). Further, the energy loss per bead for high density materials (Texture 1 and Texture 2) was typically larger than that for low density materials (Silver, 4 mm glass and 5 mm glass). To understand the effect of bead material, bead density was plotted against the minimal "distance to stop" (**Figure 5**). Since the density of the chosen bead types are in distinct narrow ranges, the data is clustered into two groups (**Figure 5**). As the density of the bead increases, the energy dissipated also increases, although investigations with beads of varying densities could further verify this observation. These observations confirm that the bead material is an important factor in energy loss.

# DISCUSSION

The data collected from the experiments contradict our original hypothesis that energy loss decreases with an increase in fill level. Initially, we assumed that the energy loss due to the collision and the friction between the beads would have a small contribution relative to the friction between the capsule and the ramp. Instead, collision and friction significantly contributed to the energy loss. There was an optimal fill level (%) for each bead type at which the energy loss was maximized. The "distance to stop" versus fill level results for all bead types were in a "U-shaped" curve, showing that if the capsule was filled too little or too much then the energy loss was less. Similarly, the energy loss within the capsule showed an inverted "U-shape", with a maximum at a certain optimal fill level.

Even when the amount of dissipation per bead was high, as was the case for low fill level, since the total number of collisions and frictional rubbing were less, the energy dissipated was low, (Figure 5). However, at low fill level, the number of beads could not dissipate enough energy to stop the capsule. At the lower fill level the energy loss was likely from collisions between beads. As the fill level was increased, the number of collisions and frictional rubbing increased, resulting in higher energy losses. Beyond the optimal fill level, the number of collisions and frictional rubbing was still more. At these higher fill levels the energy loss was likely more from frictional rubbing rather than the collisions between the beads. However, the amount of energy dissipated per bead was not enough to stop the capsule (Figure 4b). The collisions and the rubbing were likely to not be energetically intensive enough due to a lack of free space for sufficient energy dissipation. Therefore, the intermediate fill level had the greatest energy loss.

This study supported our second hypothesis that the rough texture of the beads would cause more energy loss. A qualitative measure of bead surface texture is denoted as texture index (**Table 1**). The value of the texture index was defined as five for the bead whose surface was the roughest and one for bead whose surface was smooth. The energy loss per bead was the largest for the beads with higher texture index (**Figure 4b**). Further testing must be performed

by keeping the bead material the same and varying the bead surface texture in a controlled manner.

The information obtained from the trials also supported our third hypothesis that non-spherical beads dissipated larger energy. Higher aspect ratio beads, which are less spherical, are likely to have collisions that are non-central, with bigger frictional component. The observation that beads with higher aspect ratio give higher energy loss is justified and further supports the third hypothesis (**Figure 4b**). Although the results of the experiments confirmed this hypothesis, the bead materials and diameter were not same for beads of different aspect ratios. Controlled experiments need to be performed by keeping the other variables constant and varying the aspect ratio.

Additionally, we found that the bead size and density can both affect the energy loss. However, the bead size should be considered in relation to the capsule size. For a given capsule size, the larger beads resulted in lower energy loss. Finally, it is notable that the fill level is important because it offers a way to compare across the capsule sizes. The weight and number of beads cannot be used for comparison in this case. For example, if a larger capsule is chosen, then the weight and the number of beads will be larger for a given fill level. This is the key reason why fill level has been emphasized in the discussions. However, it is notable that weight and number of beads for a given capsule size also provide insight to energy loss behavior. Further testing must be done to study the relationship between the size of the beads and the inner diameter of the capsule.

The results of this experiment can be advanced in the future by testing different bead materials and different bead and capsule sizes. The results of future investigations will provide more insight into the relationship between the bead characteristics and the inner diameter of the capsule. Furthermore, the "time to stop" could be measured in future experiments. The "time to stop" will provide information about the rate of energy dissipation for the different bead types and capsule sizes. The results presented here show that the energy dissipation within rotating capsules is complex and multi-factorial. These results provide a database that can be used by physicists and engineers to develop robust mathematical models to provide theoretical explanations of these complex phenomena and observations.

It is notable that the results of these experiments also have many real-world applications. Firstly, this experiment can be used as a simple energy loss demonstration for high school physics. Secondly, the results can be helpful for designing effective vibration control devices in buildings, bridges, and machines (4,6). Finally, many materials such as cement, pharmaceuticals, and polymers are made by mixing and grinding in cylindrical capsules (2). As a result of these experiments, engineers will be able to understand how the mixing and grinding processes can be controlled to minimize energy loss (7,8).



### **METHODS**

#### Rolling Capsule on Ramp Experiments

To conduct the experiment, a 10-degree inclined ramp with a 180cm flat ramp were used (Figure 1a-b). The capsule used in the experiments was a hollow cylinder with an internal diameter of 2.5 cm and an internal length of 7.6 cm made from plexiglass. The capsule was filled with a known weight of the selected bead. The experiments were performed with the following 6 different bead types: (1) smooth rounded composite beads formed of metal clad plastic with unknown composition, termed as "Silver" in subsequent discussion; (2) 4 mm glass beads; (3) 5 mm glass beads; (4) textured sub-rounded beads of unknown metal (denoted as Texture 1); (5) textured elliptical beads of unknown metal (denoted as Texture 2); and (6) flat beads of unknown metal (Table 1). The Silver bead is spheroidal (prolate spheroid) in shape with a cylindrical opening. The two glass beads are spherical. Bead type Texture 1 is ellipsoidal (or oblate spheroid) with a cylindrical opening along the short axis. Bead type Texture 2 is also spheroidal (prolate spheroid) in shape with a cylindrical opening along the long axis. Finally, the flat bead type is disk-like (oblate spheroid) in shape with a cylindrical opening along the short axis. The aspect ratio of each bead was determined using the formula Width/Height. A caliper was used for determining the height and width (as the maximum and minimum diameter) for each bead type (Table Further, for each bead-type, a pre-counted number of beads (10) were weighed to calculate the weight of a single bead. To determine the number of beads in the capsule, the fill weight was divided by the weight of a single bead. The independent variable for this experiment was the fill level of the capsule determined by the weight of beads. The percent fill level was defined as the percentage ratio of the bead weight at a given fill level to the bead weight at 100 % fill level. Bead density and bead size aspect ratio were also used as independent variables for analysis. The dependent variable was the stop position or the distance to stop of the capsule. The capsule, the starting position, and the ramp were kept constant during the experiment. This procedure was repeated with different fill levels for all beads. The position ("distance to stop") was measured 10 times for each fill level. After compiling the data, the average position ("distance to stop"), their standard deviations, and their coefficient of variations were found.

#### **Bead Density Determination**

The displacement method was used to find the bead density because the bead materials were unknown. A quantity of beads was weighed in the air. Then, the beads were dropped into a known volume of water contained in a graduated cylinder. The displaced water volume was found and then the equation, *density = mass/volume*, was used to calculate the density with the data that had been gathered from the displacement tests.

#### Method of Analysis

When the empty capsule is resting on the inclined ramp at its trapdoor stop, its potential energy can be calculated using the formula U = mgh (**Figure 1a**). As the empty capsule is released to rotate down the ramp, it will gain kinetic energy due to motion while losing energy due to its interaction with the ramp. As an initial assumption, the energy loss mechanism is modeled as friction loss. From Coulomb-Amonton's law of friction, *friction force=\mu N*, (1) where  $\mu$ = loss or friction coefficient and *N* = normal force, the following is obtained:

$$f = \mu mg \cos\theta \tag{1}$$

The energy loss due to friction is defined as:  $w_r = force * distance$ 

When the capsule is on the inclined portion of the ramp, the distance is given as  $d = l/cos\theta$ (3)

Therefore, the energy loss on the inclined part of the ramp is estimated as:

$$w_{f} = \mu m g l \tag{4}$$

As the capsule rolls along the flat horizontal portion of the ramp, it loses energy until it comes to a complete stop. Therefore, the initial potential energy minus the frictional loss on the inclined and the flat portion of the ramp must be equal to zero. The following equation can be found from the energy balance for an empty capsule rolling down a ramp and coming to rest on the flat ramp at distance  $L_e$ :

$$m_c gh - \mu m_c gI - \mu m_c gL_e = 0$$
 (5)

Or,

$$\mu m_c g(L_e + l) = m_c gh \tag{6}$$

Therefore, the loss or friction coefficient,  $\mu$ , between the capsule and the ramp is obtained as:

$$\mu = h/(L_e + I)$$
(7)  
nce to stop for empty cylinder = 360 cm h

where  $L_e$  = distance to stop for empty cylinder = 360 cm, h = height of the capsule in its initial position = 8 cm,  $m_c$  = mass of capsule,  $I = d \cos \theta = 45.3$  cm, d = length of the inclined ramp,  $\theta$  = ramp inclination angle = 10°, and g = acceleration due to gravity.

For the filled capsule, the loss of energy is partly from the interaction of the capsule and the ramp and partly from the bead interactions of collision and friction within the capsule. In this case, from the energy balance for the filled capsule, the following equation can be obtained:

 $W_{NC} = (m_c + m_b)gh - \mu g(m_c + m_b)(l + L_t)$  (8) where  $W_{NC}$  = Energy loss inside the capsule due to bead interactions,  $m_b$  = mass of beads,  $L_t$  = Distance to stop. This equation tells us that energy loss in the capsule ( $W_{NC}$ ) is the potential energy (mgh) minus the frictional energy loss caused by the friction between the capsule and the ramp. This equation can then be used to calculate the energy loss as a function of fill weight as well as energy loss per bead. This could give more information about how certain bead characteristics affect energy loss.

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