Determining surface tension of various liquids and shear modulus of paper using crumpling effect

Gaurav Gupta¹, Md. Laraib Salam²
¹ Kendriya Vidyalaya No.1 AFS Kalaikunda, West Bengal, India
² Air Force Golden Jubilee Institute, New Delhi, India

SUMMARY
When a wet piece of paper is dried, it becomes crumpled. We termed this phenomenon the ‘Crumpling Effect’. The crumpling effect is commonly referred to as buckling of paper by water-color artists and can be caused by water or various other liquids and solutions. We hypothesized that the surface tension of the liquid soaked in the paper acts parallelly to the plane of the sheet but perpendicularly on the paper fibers as shear stress. The shear stress of surface tension pulls the sheet inwards as an attempt by the liquid to reduce its surface energy and creates the crumpling in the sheet of paper. We experimented with different liquids and observed that the mean height of crumpling varied by liquid with an ink solution causing the greatest crumpling 0.0028 m and a soap solution causing the least crumpling of 0.0008 m. Crumpling Effect measurements can be used to determine the surface tension of any liquid absorbed by paper. However, the shear modulus of elasticity of paper is not standard and depends on the quality of materials and method of manufacture. By repeating the experiment with purified water and different paper qualities, we determined the shear modulus of the papers to be 0.043 N/m for copier paper and 0.015 N/m for bond paper.

INTRODUCTION
Paper is a matted or felted sheet made of cellulose fibers (1). A liquid that falls onto paper is absorbed by the fibers, and then occupies the space between the parallel aligned cellulose fibers (2). The force of the liquid on the paper causes shear stress, which is a force that tends to cause deformation of a material by slippage along a plane parallel to the imposed stress (3). The stress creates a permanent strain in the paper that increases the dimensions of the paper relative to its original dimensions. Hence, crumpling can be observed after the liquid evaporates and the paper becomes dry. This phenomenon is often very frequent in our day-to-day life, so we termed it as the “Crumpling Effect”. We hypothesized that the liquid present in between the fibers exerts the force of surface tension perpendicularly on the paper fibers resulting in paper crumples (Figure 1).

Based on this hypothesis, we derived a mathematical relation to relate the strains of crumpling to the surface tension of the liquid. Since the surface tension acts as shear stress, we used the mechanical shear modulus of paper to establish the relation between the shear force and strain (4). Unlike other materials such as steel, iron, or aluminum, paper does not have a definite shear modulus. It varies from sample to sample due to different paper qualities and is determined using complex methods (5). Since the shear modulus of paper is unknown, we calculated it through the crumpling effect of purified water or any control liquid (i.e. liquid whose surface tension is already known) on the paper.

Figure 1: The force of surface tension acting perpendicularly on the paper fibers. (Credit: Akash S. Balsaraf) The force of surface tension acts perpendicularly on the paper fibers and results in crumpling of paper. At the same time, the direction of surface tension force is parallel to the plane of the sheet.
phenomenon as a simple and quick test to calculate the surface tension of any liquid/solution as well as the shear modulus of elasticity of any paper type.

In our experimentation, we used purified water (as the standard test to determine the shear modulus of paper), soap solution, shampoo solution and ink solution. We obtained surface tension values for soap, shampoo, and ink solutions closer to their original value and our hypothesis was validated by the experiments. Thus, the surface tension was a cause for the crumpling effect.

RESULTS

We determined the shear modulus of elasticity of four types of paper including A4 copier paper, A4 bond paper, Classmate notebook sheet, and newspaper was determined using the derived Equation I.

<table>
<thead>
<tr>
<th>Test Liquid</th>
<th>Crumpling/Buckling Height (Δx)</th>
<th>Sample Breadth</th>
<th>Crumpling Length (l)</th>
<th>Mean Calculated Surface Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shampoo solution</td>
<td>0.001 m</td>
<td>0.0013 m</td>
<td>0.053 m</td>
<td>0.038 m 0.038 N/m</td>
</tr>
<tr>
<td>Ink Solution</td>
<td>0.003 m</td>
<td>0.0028 m</td>
<td>0.053 m</td>
<td>0.038 m 0.088 N/m</td>
</tr>
<tr>
<td>Soap Solution</td>
<td>0.001 m</td>
<td>0.0006 m</td>
<td>0.053 m</td>
<td>0.038 m 0.022 N/m</td>
</tr>
</tbody>
</table>

Table 1: The experimental shear modulus of elasticity of A4 copier paper, A4 bond paper, Classmate notebook sheet, and newspaper was determined using the derived Equation I.

The crumpling height of A4 copier paper was the lowest of all at 0.002 m and that of the Classmate Notebook sheet was highest at 0.015 m when compared to other samples (Figure 2). As a result, the A4 Copier paper had the highest shear modulus of elasticity at 0.043 N/m² and the Classmate Notebook sheet had the lowest value at 0.008 N/m² (Table 1).

Purified water was used as a standard test to determine the shear modulus of the paper sample. In the ink solution sample, the crumpling was the largest of all the samples at 0.003 m whereas crumpling was the lowest in the sample of soap solution at 0.0006 m (Figure 3). As a result, the surface tension of the ink solution was highest of all at 0.088 N/m when compared to shampoo solution, soap solution, and water. The surface tension of the liquids decreases from ink solution, water, shampoo solution, and soap solution (Table 2).

DISCUSSION

Of all the sample solutions, we observed the largest crumpling height of 0.003 m for the ink solution whereas the least crumpling height of 0.0006 m was found for the soap solution. The standard sample created using purified water showed a moderate crumpling height. Previous studies have shown the surface tension of water reduces after adding soap (7). The results of the experiment demonstrate the effect as the experimental surface tension of soap water is almost half of the purified water and the obtained value is close to the value 25 mN/m obtained in previously published research paper (7). Similarly, the value obtained for the shampoo solution is close to the mean value of 35 mN/m obtained
in another independently conducted experiment (8). The surface tension of ink comes out to be high at 0.088 N/m and accounts for the high bulge in the paper. Since the crumpling height is small, the measurements should be accurate.

The values from this experiment comply very well with the known facts and values of surface tension of test liquids determined through other methods. This supports our hypothesis that surface tension is the reason behind the crumpling effect or the buckling of paper. Hence, we can use the crumpling effect experiment as a rapid method to determine surface tensions of liquids in labs as we do with the capillary rise method. The capillary rise method works on the principle of liquid adhesive forces (9). The liquid rises inside a thin glass capillary tube and forms a meniscus (10). This phenomenon can be used to determine the surface tension of the liquid. For the experimentation, the availability of capillary tubes and a travelling microscope is extremely necessary to measure the liquid height in the tube. On the other hand, the crumpling effect experiment can give similar results by just using a sheet of paper and the liquid. Thus, the crumpling effect experiment becomes more accessible and inexpensive. Also, the experiment can be done multiple times in a short duration.

The elasticity of paper varies from sample to sample as it is dependent on the quality of paper and the method of manufacturing. Moreover, there is not any standardized method to determine the elasticity of the paper. Hence, the crumpling effect experiment can also be used as a simple and quick method to determine the shear modulus of elasticity of different paper qualities. The shear modulus of elasticity relates to the strength of the paper when any force is applied parallel to the sheet. Understanding paper strength can be effective in improving papers used in printing, converter, and packaging industries.

Since the surface tension of liquid is dependent on the temperature of the liquid and the surrounding pressure, it varies in different environmental conditions. Hence, the experiment will be repeated in a controlled manner with controlled pressure and temperature in the future which may lead to more accurate and precise results. The experiment can also be used in combination with the method to measure surface tension through capillary rise to improve accuracy. We also aim to determine how the various qualities of paper, as well as the manufacturing methods, affect the crumpling and shear stress of the sample in vast detail. We also aim to test the crumpling effect with organic solvents such as ethanol, methanol, phenol, aldehydes, carboxylic acids, as well as various inorganic salt solutions such as copper sulfate solution, acids, and bases. This will help us widen the range of the crumpling effect experiment.

With this method, we were not only able to determine the surface tension of various liquids but we were also able to easily determine the shear modulus of elasticity of paper. The other methods to determine the shear modulus of thin paper are quite difficult owing to the paper thickness (5). We can use the crumpling effect, which is presented by the thin paper, to determine the described elastic properties. Hence, determination of shear modulus of paper proves as one of the applications of the crumpling effect. In the future, we can find other applications of the “Crumpling Effect” like the ones presented in here.

**MATERIALS AND METHODS**

**Determination of Shear Modulus of Sample Paper**

A small piece was cut from the sample paper (Even though any arbitrary size or paper shape can be used to observe the phenomenon, using a small square or rectangular makes experimentation faster, and the calculations based on the experiment are easier. For example, we need the buckling length and area of the sample which is easier to measure in the case of a square or rectangular piece. Also, using small-size samples allow us to use a single A4 sheet for multiple experiments). The thickness of the sample (t) was measured using a screw gauge. Then, the sample paper was secured on the working table by taping the sample edges with masking tape. (Masking tape reduces the damage to the sample from tape adhesive). The surface was then brushed with purified water at room temperature. The sample was blow-dried for 2-3 minutes until the sample was completely dry (It can either be left for 5-6 hours under fan or can be blow-dried for faster drying). As soon as the water evaporated and the sample was dry, the paper crumpled. The process was repeated 3-4 times and the buckling height (Δx) was measured using a Vernier Caliper and was noted. It must be made sure that while measuring the height, the buckling does not depress due to excess pressure or extremely tight jaws.

By definition of shear modulus of elasticity,

\[ \text{Shear Modulus} = \frac{F_{\text{Surface tension}}}{A \cdot \Delta x} \]

Now, the force of surface tension is defined as the product of the surface tension of liquid, i.e., purified water and the crumpling length. So, mathematically,

\[ F_{\text{Surface tension}} = \sigma \cdot l \]

\[ \kappa = \frac{\sigma \cdot l \cdot t}{A \cdot \Delta x} \quad (\text{Equation 1}) \]

The area, A, of the paper is given by the product of the paper length (l) and the paper breadth, t represents the
thickness of the paper, \( \sigma \) represents the surface tension of purified water and \( \Delta x \) is the height of the crumpling/buckling (Figure 4). The crumpling height (\( \Delta x \)) is perpendicular to the direction of the stress (Figure 5).

Using the derived Equation I, the shear modulus value of the sample paper is determined. Here \( \sigma = 72 \text{ mN/m} \), \( l = \) length of the sample, \( t = \) thickness of the sample and \( A = 0.0025 \text{ m}^2 \). The method was repeated three times for different qualities of paper i.e. A4 copier paper, A4 bond paper, a sheet of Classmate Notebook, and newspaper to determine the shear modulus of each paper quality.

**Determination of Surface Tension of the Test Liquid**

For the test, we used 6 mL of Household Shampoo (Brand: Dove) and 100ml of purified water to prepare the shampoo solution. Similarly, 4ml of Generic Liquid Ink (Brand: Luxor Pilot V5) was mixed in 100 ml of purified water to prepare the ink solution and 5 ml of Dishwash Liquid (Brand: Vim) was mixed in 100 ml of purified water to prepare the soap solution. The previously described methods were repeated with each of the test liquid/solutions (Figure 6).

Again, we know

\[
\kappa \text{ (Shear Modulus)} = \frac{\text{Shear Stress} \times t}{\text{Shear Strain} \times A}
\]

\[
\kappa = \frac{\sigma \times l \times t}{A \times \Delta x} \quad \left[ \text{Surface Tension} = \frac{\text{Surface Tension} \times \text{Length} \times l \times t}{A \times \Delta x} \right]
\]

\[
\sigma \text{ (Surface tension of liquid)} = \frac{\kappa \times A \times \Delta x}{l \times t} \quad \text{(Equation II)}
\]

The measurements were made for each test liquid/solution and the previously determined shear modulus of the A4 copier paper was plugged into Equation II to obtain the surface tension of each test liquid/solution.

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**Figure 4:** Measurements made during the crumpling effect experiment with an ink solution. The length of crumpling (\( l \)), the height of crumpling (\( \Delta x \)), and the breadth of the sample marked in a sample representing the crumpling effect of ink solution on A4 copier paper were measured and further used to determine the surface tension of ink solution.

**Figure 5:** The direction of the force of surface tension of liquid acting on the sheet. The force of surface tension acts as the shear force along the surface and is perpendicular to the crumpling height in the sample representing the crumpling effect of ink solution on A4 copier paper.

**Figure 6:** Samples of Soap Solution and Ink Solution during the experiment. The sample pieces of the A4 copier were brushed with soap solution (Top, White-colored) and ink solution (Bottom, Black colored) during experimentation. The buckling/crumpling is visible in a few of the dry samples.

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**REFERENCES**


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