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Article

A model for angle evolution in conical piles formed using the fixed funnel method

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SUMMARY
Granular materials such as sand, soils, and industrial powders naturally accumulate into conical piles when poured onto a surface. A key property of these piles is the angle of repose, the maximum stable slope that a granular material can maintain, which is widely used to quantify flowability and predict the behavior of granular systems in applications ranging from soil stability and powder handling to additive manufacturing. Although the angle of repose has been studied extensively, little is known about how pile angles evolve during formation before reaching this stable maximum. We hypothesized that pile angles would increase nonlinearly and approach their final value asymptotically as material accumulated. To test this idea, we used the fixed-funnel method to construct conical piles from six granular materials spanning a broad range of particle sizes and shapes. Despite substantial differences in particle morphology, all materials followed a common two-phase exponential growth pattern in which pile angles increased rapidly at first and then more gradually as they approached the angle of repose. The parameters of this model varied among materials and correlated with internal friction estimated from particle morphology and measured angles of repose. Additional experiments showed that changing funnel diameter altered the trajectory of angle growth without substantially affecting the angle of repose. Together, these results identify a previously unrecognized quantitative framework for describing how granular piles form and provide new insight into the relationship between particle properties, deposition conditions, and pile growth dynamics.

INTRODUCTION
Granular materials—such as sand, soils, and industrial powders—display complex flow and accumulation behaviors that shape phenomena ranging from dune formation to the stability of soils, powders, and other granular deposits (1). A key property governing granular flow is flowability, or the ease with which particles rearrange and move relative to one another under an applied force such as gravity. Flowability influences how granular materials spread, accumulate, and form stable structures, making it a critical parameter for understanding both natural landscapes and engineered systems (1). One of the most widely used metrics for characterizing flowability is the angle of repose, defined as the maximum angle that a pile of granular matter can maintain relative to the horizontal plane without avalanching (1). The angle of repose provides a simple quantitative measure of the balance between gravitational forces and particle-particle interactions within a granular pile. A large angle indicates low flowability, while a small angle indicates high flowability. Several factors influence the angle of repose, including gravity, particle shape, size, density, inter-particle forces, and moisture content (2-6).

Granular piles arise in a wide range of natural and engineered systems, and their formation has been studied at different physical scales—from the aggregation of microscopic colloidal particles, to heap formation by millimeter-scale sands and meter-scale soils and geological deposits. Although these systems differ in cohesion, particle shape, and the dominant forces acting on them, they still tend to accumulate into heap-like structures shaped by friction, particle packing, and geometry (7-9). Similar behavior occurs in soil mechanics, pharmaceutical powders, and planetary regolith—the loose material covering the surfaces of the Moon, Mars, and other planetary bodies—where the balance of gravity, particle interactions, and shape effects determines heap stability and flow behavior (10). Therefore, understanding the factors that control pile formation is relevant to applications ranging from slope stability and powder handling to dune evolution and planetary regolith.

Despite the importance of pile formation, previous work studying the angle of repose has focused almost exclusively on the final static or dynamic angle formed by granular matter rather than the process by which a pile grows. For example, in a comprehensive review of angle-of-repose measurements, no relationship was identified between the mass of a conical pile and the angle it forms while sand is being poured (11). Consequently, little is known about how the angle changes as additional mass accumulates, leaving the growth rate—defined here as the change in angle as a function of mass—poorly understood. To address this gap in knowledge, we examined how the slope evolves as matter is deposited rather than simply measuring its final equilibrium value.

Three main approaches are commonly used to measure the angle of repose. The tilting-box method measures the coefficient of static friction by slowly tilting a box of grains until sliding occurs (1). The revolving-cylinder (rotating drum) method measures the dynamic angle of repose by rotating a partially filled drum and observing the flowing surface (1). This probes continuous flow rather than pile growth and yields angles smaller than the static angle because grains remain in motion. In contrast, the fixed-funnel method allows grains to fall freely and form a conical pile under gravity (1). This is the only method that enables direct, continuous tracking of slope evolution as mass increases and was therefore chosen for our study.

Using the fixed-funnel method, we studied pile formation in six granular materials with distinct particle sizes and shapes. We hypothesized that the pile's angle would increase nonlinearly before gradually approaching the static angle of

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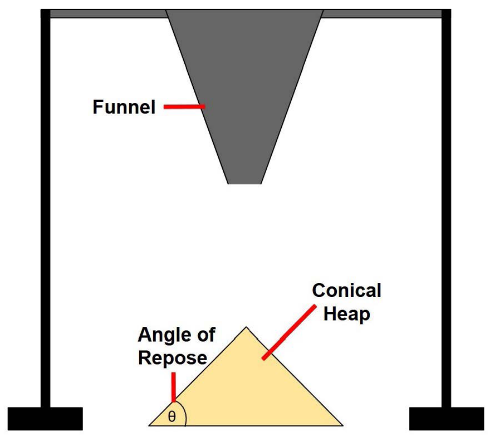

Figure 1: Formation of a conical pile using the fixed-funnel method. This schematic illustrates a conical pile formed by granular material poured from a funnel onto a flat surface. The angle of repose (θ) is shown as the angle between the pile surface and the horizontal plane.

repose asymptotically. We further hypothesized that sands with higher internal friction would exhibit faster angle growth, due to increased resistance to particle rearrangement during early deposition. These predictions were supported by our observations, and we found that angle growth in most sand piles was well described by a double-exponential function. These results have relevance for soil mechanics, powder processing, and powder-based 3D printing, where understanding how grains accumulate is essential for predicting flow behavior and layer formation (12).

RESULTS

To determine how the angle of repose evolves during granular pile formation, we monitored the accumulation of six granular materials with distinct particle properties: aluminum oxide, Florida beach sand, Zen quartz, riverbed sand, sandblasting media, and glass beads. In each experiment, granular material was deposited through a fixed-height funnel to form a conical pile, and the angle of repose (θ) was measured as a function of accumulated mass (**Figures 1-3**). The six materials spanned a broad range of particle morphologies, from highly angular particles such as aluminum oxide and Zen quartz to nearly spherical glass beads (**Figure 2**).

Despite these differences in particle shape and texture, all six materials exhibited a similar pattern of angle evolution. The pile angle increased rapidly during the initial stages of deposition and then transitioned to a slower growth regime as additional material accumulated (**Figures 4A and 4B**). This transition typically occurred within the first ~50 g of deposited material, after which angle growth slowed progressively as the piles approached their final angle of repose.

To quantitatively describe this behavior, we modeled angle growth using a double-exponential equation with two amplitude constants and two rate constants:

$$A(m) = A_1\left(1 - e^{-k_1 m}\right) + A_2\left(1 - e^{-k_2 m}\right) \qquad \text{(Equation 1)}$$

A double-exponential model was selected because the growth curves consistently exhibited two distinct phases: a rapid initial increase in angle followed by a slower approach to the final angle of repose (**Figures 4A and 4B**). The parameters A_1 and A_2 describe the contributions of the two growth phases to the final angle, whereas k_1 and k_2 describe the rates at which these phases occur.

The model provided a good fit to the experimental data for all six materials (R^2 = 0.93–0.98), with the exception of Zen quartz (R^2 = 0.87) (**Table 1**). Although all materials exhibited a similar two-phase growth pattern, the fitted values of A_1, A_2, k_1,

Figure 2: Microscopic images of the granular materials used in this study. Representative digital microscope images (3.0 mm × 3.5 mm field of view) are shown for each of the six granular materials examined. The black scale bar in the lower right corner corresponds to 1 mm.

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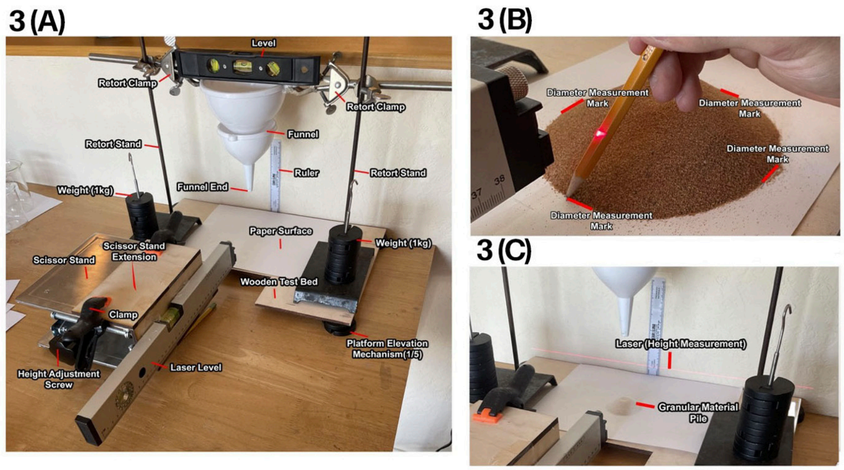

Figure 3: Experimental apparatus and measurement procedures. (A) The fixed-funnel apparatus used to generate conical piles, consisting of a funnel suspended 11 cm above a paper-covered test surface by retort stands stabilized with 1 kg weights. A laser level mounted on an adjustable scissor stand was used to measure pile height. (B) Measurement of pile diameter using pencil marks placed at opposite edges of the pile and averaged across two perpendicular directions. (C) Measurement of pile height by lowering the laser level until the beam aligned with the apex of the pile and projected onto a ruler positioned behind the pile.

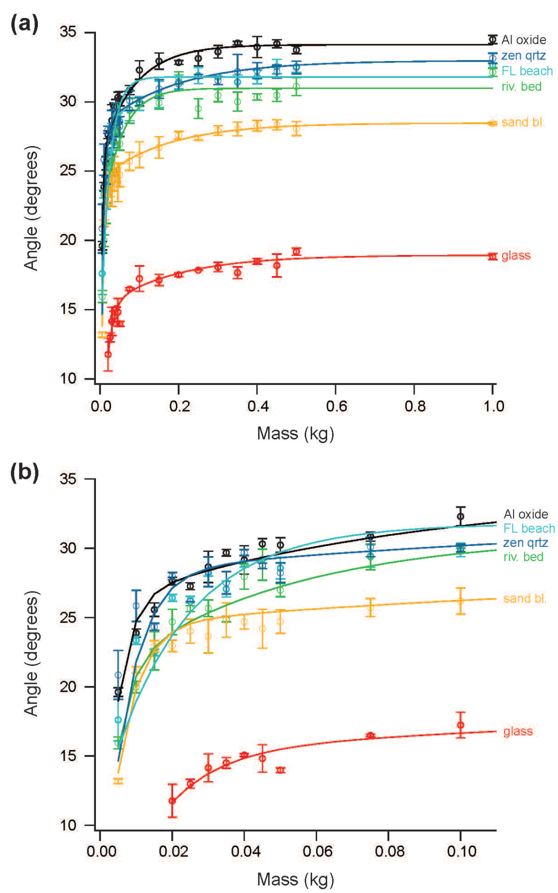

Figure 4: Angle evolution during conical pile formation. (A) Angle of repose as a function of accumulated mass for all six sands across the full experimental range (0–1.0 kg). (B) Expanded view of the initial growth phase, showing mass values between 0 and 0.1 kg. Error bars represent standard deviations from three independent trials. Curves were generated using weighted nonlinear regression, with data points weighted according to the standard deviation of replicate measurements.

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Sand Type	Tan of final angle	A1	A1 std.	k1	k1 std.	A2	A2 std.	k2	k2 std.	R^2 fit
Glass beads	0.34	15.0	0.35	69.4	4.79	3.93	0.30	5.47	1.10	0.95
Sand blasting	0.54	24.3	0.37	164	5.19	4.07	0.37	5.99	0.84	0.97
Riverbed	0.60	20.6	0.61	245	17.8	10.3	0.58	20.3	1.85	0.95
Zen quartz	0.65	28.2	0.26	144	6.09	4.75	0.31	5.32	1.00	0.87
Al. oxide	0.68	26.0	0.33	248	9.77	8.10	0.29	11.7	1.15	0.98
FL beach	0.62	11.7	0.02	784	17.4	20.1	0.02	45.4	0.06	0.93

Table 1: Parameters and quality of fit values for the double-exponential model. The table shows the static friction coefficient (tan θ), fitted amplitude constants (A_1 and A_2), rate constants (k_1 and k_2), their associated standard deviations (std), and the coefficient of determination (R^2) for each of the six sands tested Abbreviations: Al, aluminum; FL, Florida.

and k_2 varied substantially among granular materials, indicating that the magnitude and rate of angle growth depended on the physical properties of the particles. Variability among replicates remained low (<2°), indicating that the measured growth curves and fitted parameters were highly reproducible across independent trials.

We hypothesized that these differences were related to internal friction between particles. Granular materials with higher friction should resist particle rearrangement more strongly during deposition, potentially altering both the rate at which pile angles develop and the relative contributions of the two growth phases. To investigate this possibility, we examined the microscopic characteristics of the six materials (**Figure 3**).

Our microscopic examination revealed substantial differences in particle morphology. Aluminum oxide and Zen quartz consisted of highly angular particles with sharp edges, whereas glass beads were nearly spherical. Florida beach sand, riverbed sand, and sandblasting media exhibited intermediate morphologies. Based on these observations, apparent particle angularity decreased in the order: aluminum oxide > Zen quartz > Florida beach sand > riverbed sand > sandblasting media > glass beads.

To evaluate the relationship between friction and the fitted model parameters, we computed the static friction coefficient using:

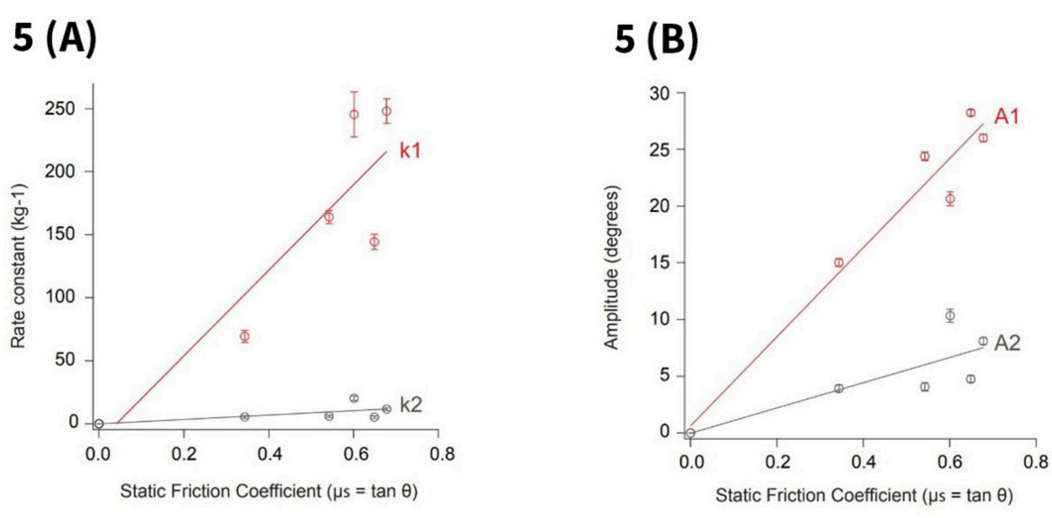

Figure 5: Relationship between model parameters and the static friction coefficient. (A) Rate constants (k_1 and k_2) plotted as a function of the static friction coefficient (μ_s = tan θ) for the five materials with the highest-quality model fits. (B) Amplitude constants (A_1 and A_2) plotted as a function of the static friction coefficient for the same materials. Error bars represent standard deviations derived from the model fits. Regression lines were generated using weighted linear regression, with data points weighted according to the standard deviation of the fitted parameters.

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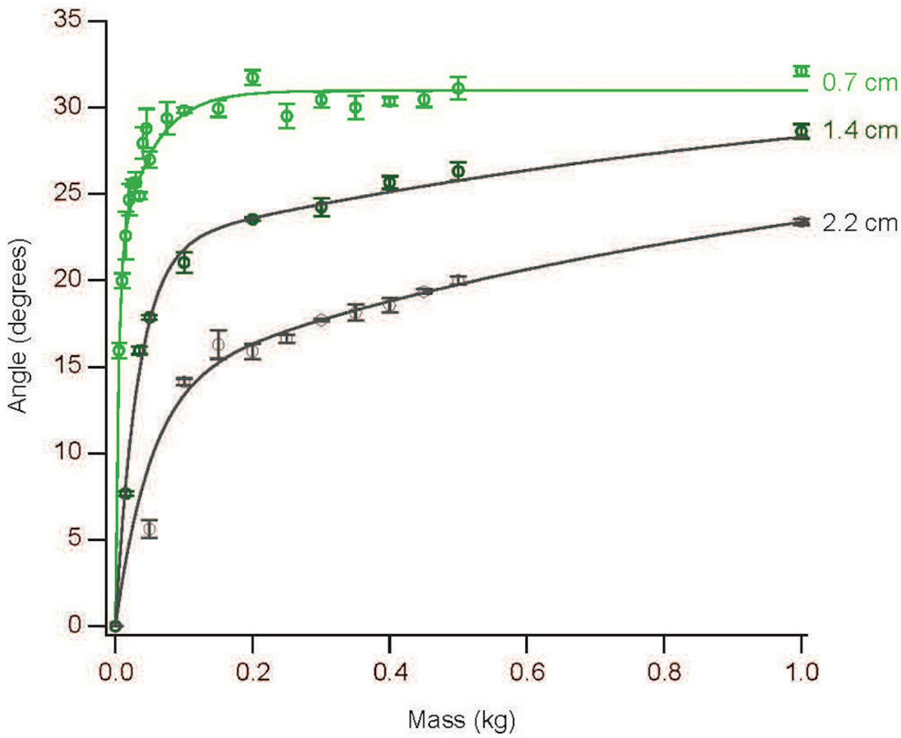

Figure 6: Effect of funnel diameter on angle evolution during pile formation. Angle of repose is plotted as a function of accumulated mass for riverbed sand deposited through funnels with diameters of 0.7 cm, 1.4 cm, and 2.2 cm. Error bars represent standard deviations from three independent trials. Curves were generated using weighted nonlinear regression, with data points weighted according to the standard deviation of replicate measurements.

$$\mu_s = \tan \theta \qquad \text{(Equation 2)}$$

This equation follows from balancing gravitational and frictional forces on a grain resting on the pile surface (13).

We found positive correlations between μ_s and each of the four model parameters (A_1, A_2, k_1, and k_2) (**Figure 5**), indicating that materials with greater friction coefficients generally exhibit larger amplitudes and faster rates of angle growth. Florida beach sand was the primary exception to this trend.

Finally, to test whether the deposition rate affected angle growth, we performed an additional experiment using riverbed sand and funnel diameters of 0.7 cm, 1.4 cm, and 2.2 cm (**Figure 6**). Riverbed sand was selected because it exhibited intermediate particle angularity and a high-quality fit to the double-exponential model.

Across all funnel sizes, the final angle of repose approached a similar value (29 to 32°) indicating that funnel diameter had little effect on the equilibrium slope. However, the early-stage growth rate (k_1) decreased substantially with increasing funnel diameter. We also observed changes in the later growth phase, indicating that deposition rate influenced the trajectory of angle growth even though the final angle of repose remained largely unchanged.

DISCUSSION

The formation of conical granular piles is a common phenomenon in both natural and engineered systems, yet most studies of granular flow have focused on the final angle of repose rather than the process by which that angle is achieved. In this study, we examined how the angle of a conical pile evolves during formation using the fixed-funnel method and tested whether this growth could be described by a simple quantitative model. Across six granular materials with substantially different particle shapes and textures, we found that angle growth proceeds in two distinct phases and is captured well by a double-exponential model. Because the final angle of repose equals $A_1 + A_2$, the model links the dynamics of pile formation directly to frictional properties through $\mu_s = \tan \theta$.

The use of a two-phase exponential model is supported by earlier granular physics studies demonstrating fast–slow relaxation processes in heap formation, compaction, and avalanche behavior (14-16). The first phase (A_1, k_1) likely reflects rapid formation of a mechanically stable base, immediately after deposition begins. During this stage, newly arriving particles can readily increase the pile angle because the pile is small and particle rearrangements occur over short distances. Once a stable base has formed, additional particles must be accommodated through progressively more complex rearrangements across a larger pile surface. The second phase (A_2, k_2) therefore reflects slower adjustments as the pile approaches its final equilibrium geometry.

This interpretation is consistent with our observation that all materials exhibited a rapid increase in angle during the first ~50 g of deposition followed by a more gradual approach to the angle of repose.

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Despite large differences in particle morphology, the double-exponential model described all six materials reasonably well. The fitted parameters varied substantially among materials, indicating that the rate and magnitude of angle growth depend on particle properties rather than being determined solely by gravity or pile geometry. In particular, materials with larger friction coefficients generally exhibited larger amplitudes and faster growth rates. This observation is consistent with previous studies showing that the angle of repose is strongly influenced by internal friction and particle shape (3-5). More angular particles are expected to interlock more readily, resisting rearrangement and allowing steeper slopes to develop during deposition.

Two notable exceptions provide additional insight into the limitations of the model. Zen quartz produced the lowest goodness-of-fit value ($R^2 = 0.87$), despite following the overall two-phase trend. Microscopic examination revealed that Zen quartz contained highly irregular, angular particles. Such particles likely generate more heterogeneous local packing arrangements and intermittent avalanching events during deposition, producing deviations from the smooth growth behavior assumed by the model. Nevertheless, the model still captured the major features of pile formation, suggesting that the two-phase framework remains broadly applicable even when particle geometry becomes highly irregular.

Florida beach sand represented the principal outlier in the friction-parameter correlations (**Figure 5**). Unlike the other materials, Florida beach sand did not follow the expected relationship between friction coefficient and model parameters. One possible explanation is its calcareous composition. Calcareous sands are composed largely of calcium-carbonate skeletal fragments and are known to exhibit particle crushing, variable stiffness, and complex mechanical behavior that differs from that of silica-based sands (17). In addition, the Florida beach sand contained a mixture of particle shapes and sizes, which may have altered packing behavior during deposition. These factors likely contributed to its atypical parameter values and highlight that friction alone may not fully explain pile-growth dynamics in heterogeneous granular materials.

Our results also suggest that the pathway by which a pile forms can be influenced independently of the final angle of repose. Altering funnel diameter changed the fitted growth constants without substantially affecting the final equilibrium angle. Larger funnel openings reduced k_1, indicating slower early-stage angle development, while changes were also observed in the later growth phase. These observations suggest that deposition rate influences how quickly a mechanically stable base forms, even though the final pile geometry is largely determined by material properties. Thus, the angle of repose appears to be an endpoint property, whereas the growth parameters contain additional information about the dynamics of pile formation.

Several limitations of this study should be noted. First, only six granular materials were examined, and the assessment of particle angularity was qualitative rather than quantitative. Second, experiments were conducted under low-humidity conditions (<20%), minimizing cohesive forces between particles. Because moisture can substantially alter granular behavior through liquid-bridge formation, the model may require modification under more humid conditions. Third, the funnel-diameter experiment was performed using only riverbed sand, so additional work will be required to determine whether the observed effects generalize across materials.

Future studies could extend this work by incorporating quantitative particle-shape measurements, broader ranges of particle size distributions, and controlled variations in humidity and flow rate. It will also be important to determine whether the model applies to cohesive powders, soils, and other granular materials beyond the sands tested here. Ultimately, a predictive framework that relates particle properties and deposition conditions to the parameters of the double-exponential model could provide a useful tool for understanding and forecasting pile formation in applications ranging from powder handling and additive manufacturing to geologic and planetary processes.

MATERIALS AND METHODS

Environmental Conditions

All experiments were performed in Tucson, Arizona, under low-humidity indoor conditions (<20%) (18). Humidity influences granular cohesion and therefore may affect pile formation dynamics. (2) Consequently, reproduction under more humid conditions may yield different results and represents a limitation of the current study.

Granular Materials and Pile Formation

Six sands spanning a broad range of particle properties—from smooth glass beads to highly angular aluminum oxide—were selected (**Figure 2**). Sand masses were measured with a 0.1 g digital scale. Most measurements began at 5 g except for the glass beads which required 15 g because smaller masses failed to form measurable piles due to excessive particle scatter. Measurements continued up to 1000 g.

The apparatus consisted of two retort stands supporting a funnel positioned 11 cm above a flat test bed (**Figure 3**). The test bed was a sheet of printer paper placed on a 0.5 cm thick wooden board. Stands were weighted with 1 kg masses to ensure stability. After loading sand into the funnel, a stopper was removed to allow free flow and pile formation.

Angle Measurements

Pile angle was calculated by:

$$angle = tan^{-1}(h/(\frac{d}{2}) \quad \text{(Equation 3)}$$

where h is the pile height and d is the pile diameter.

Height (h) was measured by lowering a laser level until the beam contacted the pile's apex and projected onto a ruler positioned behind the pile. Diameter (d) was measured using pencil marks at opposite pile edges and averaging two perpendicular measurements.

If measurements disturbed the pile, the trial was repeated. All experiments were performed in triplicate. Preliminary tests showed pile-angle discrepancies of less than 2° between replicates, indicating that three replicates were sufficient to achieve stable parameter estimation.

Between trials, the funnel, base, and glassware were cleaned using either compressed air from a blow dryer or rinsing to minimize contamination.

Model Fitting

The double-exponential model used throughout the study was defined as:

$$A(m) = A_1\left(1 - e^{-k_1 m}\right) + A_2\left(1 - e^{-k_2 m}\right) \qquad \text{(Equation 1)}$$

The data was fit to this equation using weighted nonlinear regression (in IgorPro 9, Wavemetrics, Lake Oswego, OR), with weights based the standard deviation of replicate measurements. Regression lines in Figure 5 were not forced through the origin, as $\mu_s = 0$ is a theoretical, physically unattainable condition.

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