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Article

Human comprehension of 4-dimensional rotation

Iris Reed[1], Kristen Reed[1]
[1]Lexington High School, Lexington, Massachusetts

SUMMARY

The question of whether people can comprehend four-dimensional (4D) space remains a subject of sparse scientific investigation. This research contributes to understanding the limits of human perception, neuroplasticity, and spatial reasoning. Previous research suggests that with practice, people can improve at 4D tasks, such as determining distance and angles, locating the start of a maze, navigating checkpoints, distinguishing types of motion, and judging inherently 4D properties. However, the ability to mentally rotate a 4D object has not previously been tested. We investigated whether individuals can enhance their comprehension of 4D by practicing the rotation of a 4D cube (hypercube). We aimed to examine the boundaries of human perception and the cognitive prowess in grasping concepts beyond everyday environments by utilizing various simulated models. We hypothesized that participants could improve the number of times they successfully rotated a hypercube to a target rotation in a 4D virtual environment throughout five practice sessions. We saw a trend toward improved 4D rotation after 5 practice sessions (p=0.0767). Among participants who showed early engagement—starting the experiment the same day they watched the introductory video—improvement was statistically significant (p=0.0406). These findings suggest participants may improve their understanding of 4D rotation through repeated practice. This underscores the human mind's adaptability in comprehending abstract concepts when supported by model-based approaches.

INTRODUCTION

Our eyes receive information as flat, two-dimensional (2D) images. Our brains then combine details, such as how images from each eye differ, motion parallax, and perspective, to create a sense of depth and construct three-dimensional (3D) representations of the world (1). Four-dimensional (4D) space is a theoretical extension of 3D, created by adding a fourth axis—called the w-axis (**Figure 1**). While 4D does not exist in the physical world, it can be modeled and understood mathematically.

Our brains can construct 3D representations from 2D images, however it is unclear if they can construct 4D representations using 3D input. By questioning the assumption that the human brain is limited to 3D, insight could be shed into the boundaries of human perception and the neuroplasticity of the human mind. Research into this could provide new understandings of 3D and 4D spatial reasoning and how learning 4D concepts might improve our ability to think and reason in 3D. Additionally, improving spatial reasoning skills could have applications in education, especially for STEM fields, where such spatial abilities are highly valued. Testing our ability to understand 4D could unlock new and unique approaches to complex problems and allow us access to a broad range of fascinating geometric properties beyond our current understanding. Additionally, understanding and perception of 4D could allow for easier data analysis or visualization of systems with more than three variables.

There have been few studies on 4D spatial reasoning. In one experiment, participants navigated a 4D maze projected into 3D. With repeated practice and feedback, participants successfully learned to locate the start point when they reached the end of the maze (2). Another study investigated human 4D spatial intuition using virtual reality, where participants assessed distances between vertices or the angle between edges on a hypertetrahedron (3). It was found that people could accurately judge distance and angles in 4D, but could not confirm if this was because they understood 4D (3).

Further research has investigated how individuals interpret properties specific to 4D. One study explored how participants estimated the hypervolume of 4D objects, the measure of "space" in 4D, which is analogous to area in 2D and volume in 3D (4). Results showed a significant correlation between participants' estimations and actual hypervolume, suggesting that humans can estimate 4D properties (4). In another study, participants navigated to checkpoints in a maze 4D and most achieved over 70% accuracy (5). A follow-up experiment involved participants answering questions about an N-dimensional cube with colored sides that was projected into N-1 dimensions and most performed above 70% accuracy (5). Additional studies had participants wear a virtual reality (VR) headset and then distinguish between two types of motion. These findings suggested that humans could have 3.5D perception, meaning people can extract 4D information but not fully perceive a 4D object (6).

These studies provide evidence that people can locate the start of a 4D maze, judge the distance and angle of 4D objects, estimate hypervolume, navigate in 4D space, make judgments about N-cubes, and differentiate between types of motion. To date, no studies have been conducted to test whether people can understand and improve their rotation of 4D objects. Rotation in 4D is complex because there are six rotation planes because six planes can be made from combinations of two of the four axes. Rotation is increasingly complicated in higher dimensions because the order of

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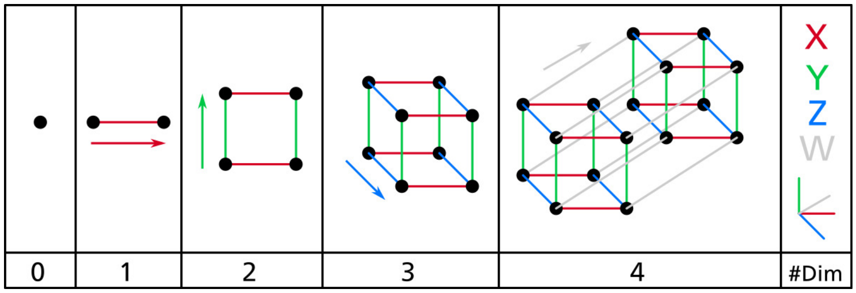

Figure 1. Diagram showing 0D, 1D, 2D, 3D, and 4D. Created by NerdBoy1392 under Creative Commons license CC By-SA 3.0. Accessed 17 Aug 2025 from https://en.wikipedia.org/wiki/Dimension.

rotation matters (rotation is non-commutative) and the number of rotation planes grows quadratically (½(N^2-N) or N choose 2) (7). Prior studies have shown humans can improve their 3D spatial reasoning abilities and mental rotation skills through training, suggesting that similar improvements may be possible in 4D spatial reasoning and mental rotation (8,9). We aimed to investigate the possibility of individuals enhancing their comprehension of the rotation of 4D hypercubes. We hypothesized that participants would improve their ability to rotate a hypercube in a 4D virtual environment over five practice sessions, as measured by the number of successful rotations. Two virtual environments, one in 3D and one in 4D, were used for the task of rotation-matching. We found that participants made a distinct improvement in their ability to rotate a hypercube, suggesting that people can understand 4D. Four participants stood out for their high performance on the 4D rotation task. Additionally, participants' performance in 3D rotation correlated with their 4D performance, suggesting that spatial reasoning skills played a key role. Participants who demonstrated more engagement in the study, as measured by whether they started the experiment the same day they watched the initial video, performed better in both 3D and 4D rotation. Our results emphasize the flexibility of the human mind to comprehend abstract concepts aided by a model-based approach.

RESULTS

Throughout five experimental sessions within a 10-day period, 20 participants completed two tasks per session. First, they rotated a 3D cube in the 3D virtual environment to a target rotation as many times as they could for 10 minutes (**Figure 2**). When they reached the target orientation, the cube reset, and they were shown a new target orientation. Then, participants performed the same task with a 4D hypercube in the 4D virtual environment. The sessions were completed remotely; neither the location nor the time were controlled. Participants also completed a pre-survey and a post-survey. Participants who did not complete all five sessions or finish within 10 days were excluded from the data.

Overall results

Participants increased their number of completed rotations in both 3D and 4D over the five experimental sessions. The average number of 3D rotations had a strong positive correlation with the session of the experiment (R^2=0.971, slope=2.08, **Figure 3**). The average number of 4D rotations had a weak positive correlation with the session number of the experiment (R^2=0.382, slope=0.1, **Figure 4**). In addition, we found that participants' improvement from session 1 to session 5 was statistically significant for 3D rotation but not significant for 4D rotation. For 3D rotation, the mean ± SD of the difference of rotations was 8.55 ± 8.217 (one-sample t-test, p=0.000087, **Figure 3**). For 4D rotation, the mean ± SD of the difference of rotations was 0.55 ±1.63 (one-sample t-test, p=0.0748, **Figure 4**).

Four participants stood out as particularly successful at 4D rotation. While 16 participants completed between 0 and 1.2 4D rotations, these four completed 2.8 to 3.2 4D rotations on average (**Figure 5**). There is no clear reason why these participants performed better. They ranged in age from 17 to 55, in level of education from high school to PhD, and included both men and women, representing the more

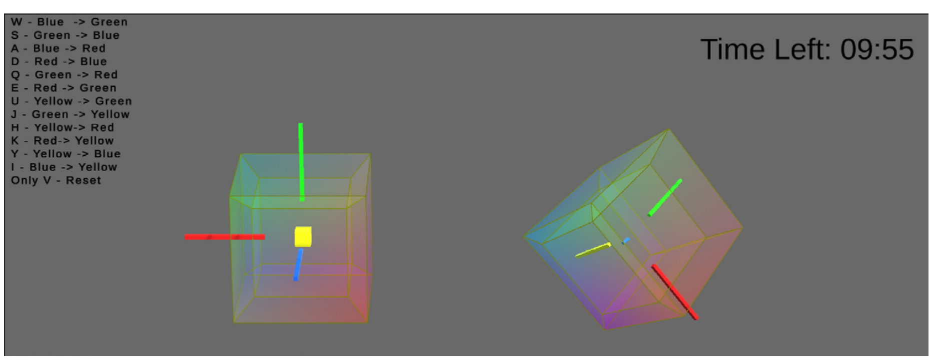

Figure 2: 4D Virtual Environment. On the right is a hypercube at a random rotation; this is the target orientation. On the left is the hypercube that the participant rotates with the keyboard inputs. On four perpendicular sides of the hypercube are colored rods to help with rotation direction - the red, green, blue, and yellow rods represent the x, y, z, and w axes, respectively - and distinguish the 192 rotationally symmetrical orientations. On the top left is a key indicating what each key does. On the top right is the time left until our program stops and shows the results.

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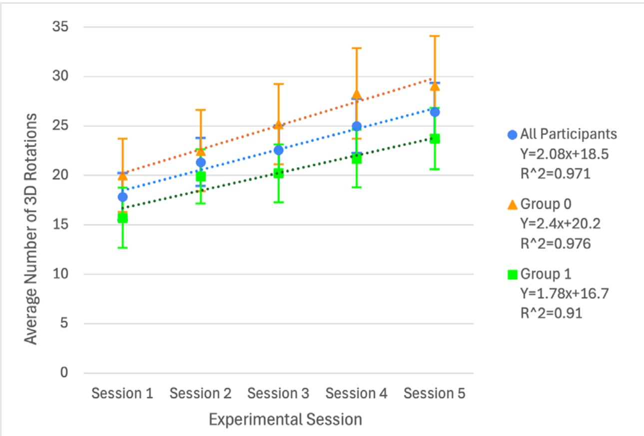

Figure 3. Average 3D rotation completions by group. Average number of completed 3D rotations in the 3D virtual environment for 10 minutes over the five sessions for all participants (blue) with a least squares regression line (LSRL) (R^2=0.971, slope=2.08). The more engaged group (orange), the participants who completed session 1 the same day they watched the initial video, exhibited an LSRL (R^2=0.976, slope=2.4). The less engaged group (green), consisting of participants who completed session 1 a day or more after watching the initial video, showed an LSRL (R^2=0.91, slope=1.78). Data shown as mean ± standard error.

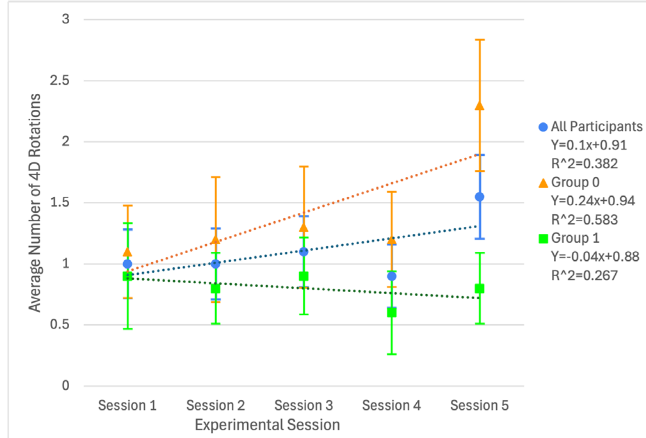

Figure 4: Average 4D rotation completions by group. Average number of completed 4D rotations in the 4D virtual environment for 10 minutes over the five sessions for all participants (blue) with an LSRL (R^2=0.352, slope=0.1). The more engaged group (orange) is the participants who did session 1 the same day they watched the initial video with an LSRL (R^2=0.583, slope=0.24). The less engaged group (green) is the participants who did session 1 a day or more after they watched the initial video with an LSRL (R^2=0.267, slope=-0.04). Data shown as mean ± standard error.

engaged group and the less engaged group. Three of the four participants performed above an average level on the 3D rotation task, while one performed below average. In the final survey, they reported their increase in understanding of 4D rotation on a scale of 1 (no increase) to 5 (large increase), these four participants reported scores of 2, 3, 4, and 5. This range reflects varying self-perceived gains among the high-performing group (compared to the overall average of 2.48 ± 1.29 across all participants). The four high-achieving participants had a strong positive correlation with the session number and the number of 4D rotations (R2=0.655), while the rest of the participants had a very weak correlation (R2=0.184, **Figure 5**). The four participants also completed an average of three rotations—4.7 times more than the rest, who averaged 0.6375 rotations. Additionally, the high-achieving participants increased more in successful 4D rotations throughout the experiment than the rest of the participants (slope=0.3 and slope=0.05, respectively, **Figure 5**).

Participants rated their computer comfort on a scale of 1 to 5 (range: 3–5). Participants' comfort correlated with better average 3D and 4D performance (R^2=0.99 and R^2=0.797, respectively, **Figure 6**). Participants with comfort levels of 3, 4, and 5 on computers completed on average 0.4, 1.15, and 1.2 4D rotations, respectively. A similar pattern was observed in 3D rotation (9.3, 18.9, 25.6, respectively). This suggests that greater computer confidence led to more improvement.
There was no correlation between age and average 4D rotations, change in 4D rotation, average 3D rotations, or change in 3D rotations (R^2=0.006, R^2=0.05, and R^2=0.165, R^2=1.01, respectively).

Attention and engagement

Participants in the more engaged group (participants who started directly after watching the video) and the less engaged group (participants who waited one or more days after the initial video to begin) both have a strong positive correlation between the session and the average number of 3D rotations (R^2=0.976 and R^2=0.91, respectively, **Figure 3**). However, the more engaged group completed on average 4.78 more 3D rotations than the less engaged group. Additionally, the rate of improvement was higher in the more engaged group, with a slope of 2.4 compared to 1.78 in the less engaged group (**Figure 3**).

Comparing 4D rotations, the contrast between the two groups was even more stark. For the more engaged group, there was a moderately strong and statistically significant positive correlation between the session of the experiment and the number of 4D rotations (R^2=0.583, slope=0.24, p=0.00508, **Figure 4**). In contrast, for the less engaged group, there was a weak negative correlation between the session of the experiment and the average number of 4D rotations, which is not statistically significant (R^2=0.267, slope=-0.04, p=0.0631, **Figure 4**). Additionally, on average, participants in the more engaged group completed 0.6 more 4D rotations than participants in the less engaged group. Interestingly, while for the overall sample, improvement in 4D was not significant with p=0.00748, when including just the ten participants in the more engaged group, the improvement from session 1 to session 5 was significant (one-sample t-test, p=0.0406, **Figure 4**). The distributions of the difference in rotations for the more engaged group and the less engaged group are approximately normal. Those in the more engaged group showed significantly more improvement in the number of rotations from session 1 to session 5 (**Figure 7**).

Performance in 3D correlated with performance in 4D

Results show a moderately weak but statistically significant positive correlation between the total number of 3D and 4D rotations (R^2=0.369, slope=0.0571, p=0.0045, r=0.608, **Figure 8**). There is a weak positive correlation that is not significant between participants' improvement in 3D rotations (from session 1 to session 5) and their improvement

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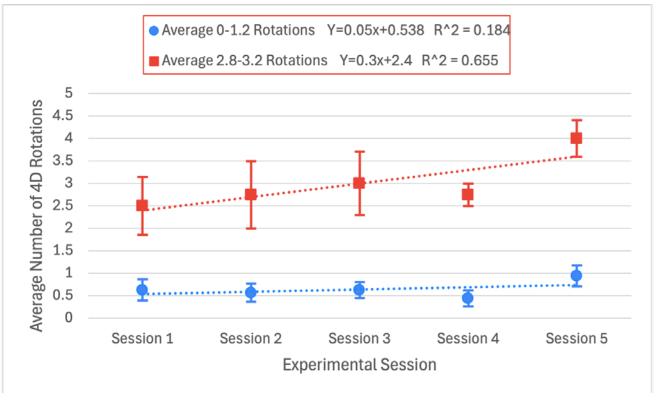

Figure 5: Number of 4D rotations grouped by 0–1.2 average rotations and 2.8–3.2 average rotations. Analysis of participants who performed better at 4D rotation with the rest of the participants. Average number of 4D rotations over the five experiment sessions for two groups: participants who averaged 0–1.2 4D rotations with 16 participants (blue) and those who averaged 2.8–-3.2 rotations with 4 participants (red). Participants who averaged 0–1.2 4D rotations are represented by the blue LSRL (R^2=0.184, slope=0.05). These participants showed minimal improvement. The average number of 4D rotations over the five experiment sessions for participants who average 2.8–3.2 4D rotations is represented by the red LSRL (R^2=0.655, slope=0.3). Data shown as mean ± standard error.

in 4D rotations (R^2=0.192, slope=0.0875, p=0.0535, r=0.4379, **Figure 9**). Additionally, when looking at the more engaged group specifically, there is a moderately strong and statistically significant positive correlation between the number of 3D and 4D rotations (R^2=0.666, slope=0.0732, p=0.00397, r=0.816). This group also has a moderately weak positive correlation between the change in 3D and 4D rotations, which is not significant (R^2=0.313, slope=0.107, p=0.0923, r=0.559).

DISCUSSION

We aimed to explore whether humans could enhance their understanding of 4D, specifically the rotation of a 4D hypercube as represented in a 2D computer-based virtual environment built specifically for this experiment. Our results suggested that participants could rotate a hypercube to a target rotation and that they improved with practice. To test the validity of the 4D virtual environment, participants also rotated a 3D cube. Results showed that participants successfully used the environment in 3D and improved the number of 3D rotations across five experimental sessions. For 4D rotation, results suggested that people can improve with repetition, as there was a moderately positive correlation between the session day and average completed rotations. Although this trend was not statistically significant (p=0.0748), it suggests potential for learning over time. Future research with a larger sample size and better control of confounding variables may yield more conclusive insights.

The strong performance of the four high-achieving participants suggests that some people may have a particular proclivity for 4D rotation. However, there is no clear indicator that explains why these four participants performed better. Interestingly, other studies reported similar individual-level differences in 4D task performance but found no clear indicator (2,3,6). Future research could further explore certain characteristics, like spatial reasoning ability or experience with math, that may contribute to this advantage at 4D tasks.

Researchers have hypothesized that people's attention and motivation may influence 4D task performance, with some people needing different types of training or longer periods to succeed (1,5). In the analysis phase, we explored whether beginning the experiment on the same day as watching the initial video could serve as a proxy for engagement. We hypothesized that the delay in start time may have been related to participants' amount of free time, their level of effort, and/or their interest in the experiment. We divided participants into two groups during the analysis of the data: the more engaged group (who began on the same day) and the less engaged group (who delayed by at least one day). The more engaged group significantly improved in their 4D rotations over five sessions, compared to the less engaged group, suggesting increased engagement improves participants' ability to learn 4D rotation. The overall sample (both groups combined) showed improvement, but it was not statistically significant with p=0.0748. While attention and engagement seem a likely explanation for the difference between the two groups, it is also possible that by practicing rotation in 4D immediately after watching the video, the more engaged group participants gained an advantage over the less engaged group participants. In future work, we could ask participants about their level of interest and amount of free time before the experiment begins, and we could randomly assign some participants to watch the video on the same day

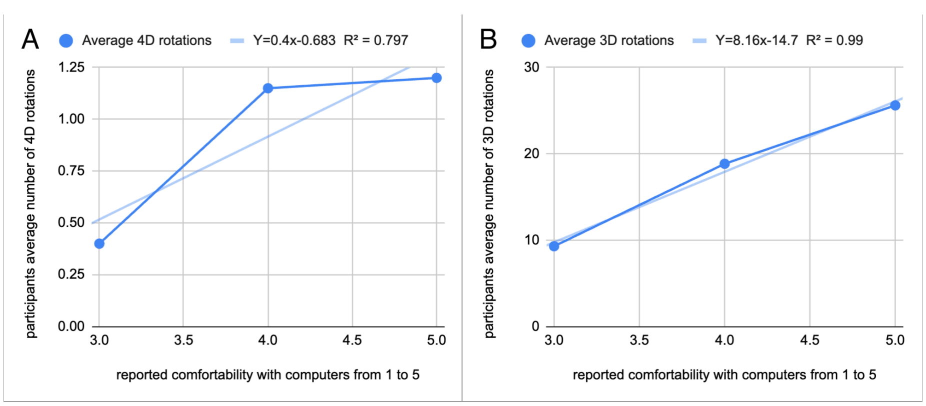

Figure 6: Participants' reported comfort with computers and their average number of 4D and 3D rotations. A) Average number of completed 4D by reported comfortability with computers with a least squares regression line (LSRL) (R^2=0.797). **B)** Average number of completed 3D by reported comfortability with computers with a least squares regression line (LSRL) (R^2=0.797).

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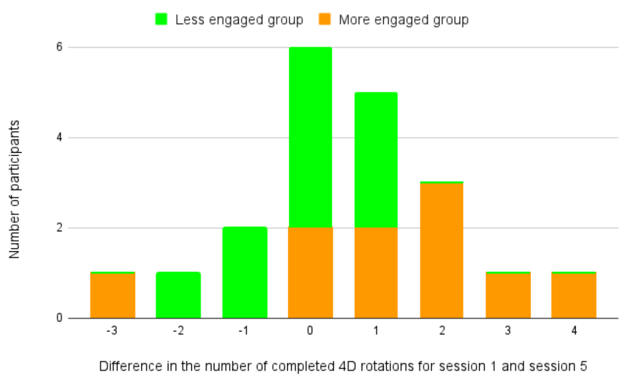

Figure 7: Difference in the number of completed 4D rotations from session 1 to session 5 by group. The number of participants and their individual differences in the number of completed 4D rotations between session 1 and session 5. A negative difference indicates that a participant completed fewer 4D rotations at the end of the study compared to the beginning, and a positive difference indicates that the participant completed more at the end of the study compared to the beginning. Results are shown for all 20 participants, as well as separately for the more engaged and less engaged groups. The data follows a nearly normal distribution across all 20 participants (One-sample K-S test, $D(20)=0.19$, $p=0.4$), the 10 participants in the more engaged group (one-sample K-S test, $D(10)=0.17$, $p=0.9014$, and the 10 in the less engaged group (one-sample K-S test, $D(10)=0.26$, $p=0.435$.

as they begin the experiment, and others to begin the day after viewing the initial video.

Participants' performance in 3D correlated with their performance in 4D, suggesting transferable spatial skills. While these results are promising, this experiment does not determine how 3D reasoning supports 4D reasoning. The significant correlation between 3D and 4D performance and the correlation between the improvement of 3D and 4D rotations approach significance, suggesting that spatial reasoning ability was a factor in participants' progress. In the more engaged group, a moderately strong correlation between 3D and performance, combined with the near-significant correlation between changes in 3D and 4D rotations, suggests that participants' 3D spatial reasoning ability influenced their performance in 4D rotation when they were actively engaged. Future research could include cognitive assessments (e.g., spatial reasoning, abstract reasoning, and working memory tests) to investigate potential correlations.

Prior research has suggested a possible critical period for developing 4D spatial reasoning (2). It is possible that developmental experiences—such as playing certain sports or video games, or an interest in geometry or maps—may strengthen spatial reasoning skills, allowing participants to better 4D spatial reasoning later in life. While participants' ages ranged from 17 to 80, there was no significant correlation between age and average 4D rotations, change in 4D rotation, or average 3D rotations. However, more data, especially focusing on younger participants, could help clarify whether 4D spatial reasoning is more malleable in childhood. Our study has several limitations. First, the experiment took place over only five sessions, which may not be sufficient for most people to develop 4D rotational ability. Notably, performance dipped in session 4 before rebounding in session 5, leaving it unclear whether participants would have continued improving, experienced another dip, or shown greater progress with additional sessions (**Figure 4**). With only five data points, outliers could skew the results and increase variability, reducing the certainty of the findings. Second, inconsistent gaps between participants' experiment sessions (0 to 5 days) may have hindered steady improvement, as practicing daily might have led to more consistent progress. Third, uncontrolled environmental factors (time of day, location, mental state) may have increased the variability in participants' performances. Additionally, participants were a convenience sample, so confounding variables, like education level, were not controlled for, limiting generalizability.

Aspects of the virtual environment may have influenced the results. The 3D rotation task required 6 keyboard inputs, while the 4D task required 12, making the controls more complex, particularly for those who do not regularly use keyboard inputs. Future studies should account for participants' familiarity with the environment by establishing a baseline pre-test or improving the interface for better usability (i.e., color coding, better keyboard layout with pairs of keys on the top and middle rows). Additionally, the virtual environment was programmed to randomly generate a reference cube or hypercube, but certain orientations are more difficult to reach than others. Pre-programming standard target orientations across participants would improve reliability. The virtual environment generated random target orientations, but some are harder to reach than others—especially in 4D, where participants completed so few rotations that one difficult orientation could consume the entire session. A key challenge in studying 4D perception in a 3D world is determining whether participants truly understood 4D space, are adapting to task-specific cues, or are gaining familiarity with the virtual environment. Future experiments could test different virtual environments with varying controls or 4D models to see if learning 4D transfers to a new system. Additionally, data could be collected on the closest rotation, the number of inputs, the length of inputs, and the time between inputs to better analyze how the participants matched the target orientation, such as seeing if they have a consistent strategy or get stuck.

There are several challenges to studying human perception of 4D in a 3D world. Participants were given feedback when they completed a 4D rotation in the form of a new random rotation. While helpful, this may have encouraged participants to pick up on low-level cues, such as the position of a colored rod, rather than deepen their understanding of 4D. Participants may have repeated their previous successful behavior without fully understanding the underlying concepts of 4D rotation (10). In other words, rather than improving their understanding of 4D rotation, participants may have been getting better at moving the colored rods to the correct location (the colored rods projected perpendicular to the faces of the cube and hypercube to indicate rotation direction). Future experiments could structure sessions without immediate feedback to assess comprehension more rigorously. For example, practice sessions could be broken into shorter trials in which participants work on a rotation until they feel confident or reach a time limit, and our program would record their time till completion, how close they came, and their average distance to the target rotation without providing feedback. Some prior research into 4D perception has been done with participants receiving no feedback and found that people could perform 4D tasks (3,4). To eliminate

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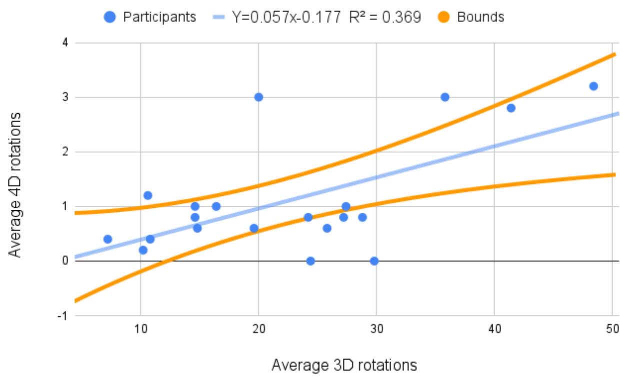

Figure 8: Average number of 3D rotations compared to the average number of 4D rotations. Comparison of the average number of 3D rotations compared to the average number of 4D rotations for each participant with an LSRL (R^2=0.369, slope=0.057). The lower and upper bound represents the 95% confidence interval of the LSRL (p=0.0045, r=0.608).

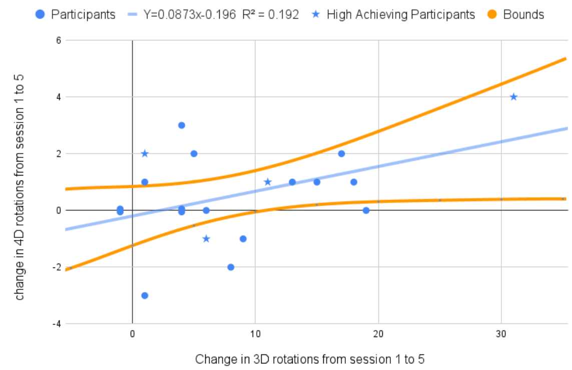

Figure 9: Change in the number of 3D rotations compared to the average number of 4D rotations. Comparison the change in 3D rotations from session 1 to session 5 compared to the average change in 4D rotations from session 1 to session 5 for each participant with an LSRL (R^2=0.192, slope=0.0873). The lower and upper bound represents the 95% confidence interval of the LSRL (p=0.0538, r=0.608). The high-achieving participants are marked with a star.

the problem with the colored rods, researchers could test alternative visualizations where the edges of the hypercube are all a different color, so you are representing 4D space while keeping the orientation distinct.

One of the most exciting aspects of our research is that there are so many questions left to be explored. First, what 4D models are easiest for people to understand? Second, do participants understand 4D, or are they just getting better at the specific virtual environment? Third, how are participants' spatial reasoning skills related to their ability to perform tasks in 4D? Many of these questions can be applied to other areas of 4D perception that have been studied, such as 4D path integration, judging distance in 4D, judging angle in 4D, judging hypervolume, navigating in 4D space, making judgments about N-cubes, and distinguishing between rotational and deformational motion. They could also be applied to areas that have not yet been studied, like the mental manipulation of 3D nets into 4D, the mental Boolean operation of 4D objects, predicting the cross-section of 4D objects when looking at a projection, and predicting projections of 4D objects when looking at a cross-section .

Overall, our research provides evidence that people can improve at 4D rotation with practice. While statistically significant improvement for all participants was not observed, those with higher engagement showed clear progress. Spatial reasoning ability correlated with 4D performance, suggesting that higher-dimensional spatial skills may be an extension of 3D spatial skills. The presence of high-performing participants indicates individual variability in 4D aptitude, warranting further investigation. Prior research has shown that people can perform and improve at other 4D tasks (e.g., path integration, judging hypervolume, distinguishing between rotational and deformational motion); however, this is the first study to test and find evidence that people can improve at rotating 4D objects (2-7). Because 4D rotation requires understanding six independent rotation planes and complex spatial relationships, the ability to improve suggests the development of internalized 4D representations. Future research should refine experimental methods, explore alternative 4D models, and investigate cognitive factors contributing to success in higher-dimensional reasoning.

MATERIALS AND METHODS

We recruited a convenience sample of 22 participants, 20 of whom completed the experiment. These 20 ranged in age from 17 to 80 M=49.7±14.1, level of education from high school to PhD, and job types. Seven were women and 13 were men.

Before beginning the practice trials, participants signed the informed consent form, which outlined the study purpose and procedures, the risks and benefits of participation, and the time requirements. After signing the consent form, participants took an initial survey and were asked to download the 3D and 4D virtual environments on their personal computers (**Appendix A**). Then, before beginning the experimental sessions, participants were instructed to watch a video that provided background information about how to understand four dimensions, what rotation looks like in a 4D, the keyboard inputs, and the experimental procedures (**Appendix B**) (11-14).

Participants were instructed to engage in five experimental sessions—one per day—within a 10 day period, allowing flexibility to skip a day if needed. We hypothesized that five sessions would give participants enough time to improve at 4D rotation while keeping the study convenient and flexible (2). Participants decided when to engage in the practice sessions independently. During each session, participants performed a 3D rotation for 10 minutes, and the program automatically recorded the timestamp for each completed rotation. Next, they performed a 4D rotation for 10 minutes, with timestamps similarly recorded for each completed rotation. At the end of each session, participants filled out a Google Form Survey, which included their participant number, the session number (sessions 1 through 5), and the timestamps for each completed rotation (**Appendix C**). After their fifth session, participants completed a post-survey (**Appendix D**).

However, participants varied in how closely they followed one of the key procedures. Half of the participants (the more engaged group; n=10) completed their first experimental session the same day they watched the initial video (following the study procedure), while the other half (the less engaged group; n=10) waited a day or two, and this difference in procedure showed relevance in our final analysis, as described in the results section. The two groups had similar background characteristics, like age, gender, and career. The more engaged group included four women and six men; age: M=47.6±18.5 years. The less engaged group included three women and seven men; age: M=51.7±8.2 years.

3D and 4D virtual environments

Using the game development engine, Unity, and Engine4, a Unity extension for making games in 4D, we created two virtual environments—one in 3D and one in 4D (**Appendix B**) (15). The setups of the 4D and 3D virtual environments were the same. The 4D virtual environment shows two hypercubes, while the 3D virtual environment shows two cubes (**Figure 2**). The cube or hypercube on the right is fixed, serving as the target orientation. Using keyboard inputs, participants rotated the cube or hypercube on the left to match the target orientation. On three sides of the cube and four sides of the hypercube, there were colored rods. The colored rods were used to indicate rotation direction and distinguish the rotationally symmetrical orientations (in N dimension, there are *2N-1*N!* rotationally symmetrical orientations of an N-cube) (7). On the top left of the screen, there was a key explaining how each button press rotates the cube or hypercube, from one colored rod to another. Pressing "v" resets the rotation cube or hypercube on the left to its starting position. On the top right of the screen was the time remaining. In 3D, six keys (A-D, W-S, and Q-E) allow rotation around three planes. In 4D, 12 keys (A-D, W-S, Q-E, H-K, U-J, and Y-I) enabled rotation around six planes.

Statistical tests

To evaluate participant improvement across sessions, we used one-sample t-tests to compare the change in the number of successful rotations in session 1 and session 5 for the 3D tasks (all participants) and 4D tasks (all participants), and the 4D tasks for participants who were more engaged and less engaged. Linear regression analyses were conducted to assess the relationship between session number and average number of rotations across all five sessions. Additional regressions examined participants' age and comfort with computers against their performance in 3D and 4D, and well as the relationship between their 3D and 4D performance. For each regression, R^2 values, slopes, and p-values were reported. Comparisons between subgroups (e.g., more vs. less engaged participants) were also analyzed using separate linear regressions. To assess whether the distributions of change scores in 4D rotation were approximately normal, we conducted the Kolmogorov–Smirnov test for normality. Pearson correlation coefficients (r) were calculated to measure the strength of the relationship between 3D and 4D performance, both for total rotations and for improvements over time. Statistical significance was set at $p < 0.05$. All statistical analyses were conducted using the full dataset from participants who completed all five sessions within the 10-day period.

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APPENDICES

Appendix A
The initial survey given to the participants.

What is your participant number? (for confidentiality)
How old are you?
What previous experience do you have with 4 Dimensions and 4D rotation (if any)?
What is the highest math class you have taken?
What is the highest degree you have?
What is your career (i.e. student, engineer)? Does it involve math or spatial reasoning skills?
Do you play video games and/or have experience with using a computer?
How comfortable are you with using a computer? (1-Not Comfortable, 5-Very Comfortable)

Appendix B
Code Repository: https://github.com/YumYumYucky/4D_Rotation
Initial Video: https://drive.google.com/file/d/1y56FmrWcyftbqdK-3mPb2oCEZINfAiBs/view?usp= sharing

Appendix C
A copy of the daily survey given to the participants is shown below.

What is your participant number? (for confidentiality)
What day of practice is it for you?
- 0. 1
- 0. 2
- 0. 3
- 0. 4
- 0. 5

What were your times for 3D rotation today? (copy and paste)
What were your times for 4D rotation today? (copy and paste)

Appendix D
A copy of the post-survey given to the participants is shown below.

What is your participant number? (for confidentiality)
Has this experiment increased your understanding of 4D rotations? In what way has it increased your understanding?
Has this experiment increased your understanding of 4D rotations? In what way has it increased your understanding? (1-Did not increase understanding, 5-Understanding increased dramatically)
Did you watch any additional videos?
- 0. Yes
- 0. No

Was there a particular way of thinking about 4D or a method that helped you out? What was it?
What changes might you make to the experiment? How would these changes help you learn?
Any other thoughts for feedback about the experiment process?
Did you do day 1 of the experiment directly after you watched the video? If not, how long was it between when you watched the video and day 1?