JOURNAL OF
EMERGING INVESTIGATORS

Article

Risk-adjusted return measures for selecting optimal mutual fund investment portfolios

Nikhil Gangavarapu[1], Susan Brontman[1], Venkat Gangavarapu[1]
[1] Adlai E. Stevenson High School, Lincolnshire, Illinois

SUMMARY

Investing has become an increasingly popular way to generate passive income. The purpose of this study was to determine which combination of the risk-adjusted return measures of Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha would provide the investor with the optimal investment portfolio of mutual funds to maximize returns. Using data from 1995 onwards, mutual funds were compared against the Dow Jones Industrial Average to evaluate relative performance. We hypothesized that the portfolio would perform better as more risk-adjusted return measures were used in conjunction because each individual measure already attempts to find the best mutual funds to use for the investor. Therefore, by combining various measures, there would be a multitude of supposed best performers that would provide greater returns. The results indicated that for shorter timeframes of 5 and 10 years, the M2 measure delivered the highest returns of 3.65% and 17.78%, respectively. Over longer periods of 15 and 20 years, Jensen's Alpha outperformed, with returns of 52.57% and 125.38%, respectively. However, for more risk-averse investors, the combination of M2 and Jensen's benefited from 116.12% returns and a larger number of funds, increasing diversification for slightly less profits. This study highlights the value of testing and refining the methods used to evaluate investments. By demonstrating that some measures excel in specific timeframes or risk preferences, it encourages investors to critically assess and adapt their strategies. offering a framework for informed decision-making in an unpredictable market, thus benefiting both individual and institutional investors.

INTRODUCTION

The stock market has been an emerging power in the lives of Americans for decades, but in recent years, it has reached unprecedented levels of investments (1). As a way to invest money, the stock market provides people with a method to supplement their income or increase their savings. According to The Washington Post, 61% of Americans have invested in the stock market (2). However, the market does not come without its risks, which is why many different methods have been devised to help a person navigate the variety of securities and choose which funds will form their investment portfolio.

A sizeable amount of the money invested by Americans is invested into mutual funds. In 2020, about 46% of US households invested in mutual funds (3). The average number of mutual funds held by each household was approximately four with varying amounts of money invested depending on the household's income (3). Furthermore, nearly 27 trillion dollars in total net assets were invested in mutual funds in 2021 (1). In comparison, the total US market value in 2021 was about 52.3 trillion dollars, meaning mutual funds accounted for almost 52% of total investments (1). Mutual funds are not the same as regular stocks, where an individual simply purchases and individually holds a share of ownership in a company (4). Instead, with a mutual fund an individual pools their money with other investors to "mutually" buy stocks, bonds, or other securities (5). Mutual funds are all looked over by professional fund managers who are guided by disciplined rules and are typically more objective when making investments, thus lowering the risk compared to if an investor decides to invest independently (6).

There are two types of mutual funds: index funds and actively managed funds. Index funds track an index, a benchmark bundle of predetermined stocks, and are primarily focused on gaining market average returns by copying that index instead of trying to beat it (7). Index funds are therefore a passive fund, as the fund tends to be automated to exactly match the securities that the index holds. Index funds are also more predictable and are a safer investment (7). Actively managed funds, on the other hand, are more aggressive and seek to beat the market, thus making a greater than average return (8). However, actively managed funds are uncertain and riskier, as they are subject to human error because there are stock pickers that actively choose what securities the money will be invested into. One more key difference between the two funds is the managerial fee that they charge. Since index funds are passive, the expense ratio, or the operational costs, has an average of only 0.06% (7). However, actively managed funds charge a much higher average fee of 0.47%, which is nearly eight times as much as index funds (7).

Among mutual funds, large growth funds are classified as funds that invest a large amount of their assets into companies with a large market capitalization (large cap), which is the value of a stock price multiplied by total outstanding shares (9). Large-cap stocks are typically from blue-chip companies, which are well-established and financially sound, with common examples being Alphabet, Microsoft, or Meta (10). For an investor, the risk is significantly reduced when investing in large-cap stocks compared with mid-cap and small-cap stocks. Since mutual funds hold numerous stocks, less risky stocks result in less risky mutual funds.

https://doi.org/10.59720/24-016

The capital asset price modeling (CAPM) is one of the most widely studied and used investment pricing models. CAPM compares systematic risk and the expected returns of investment by establishing a linear relationship between the two (11). However, the CAPM does not account for many flaws in its usage, such as making unrealistic assumptions and relying on a linear interpretation of risk vs. return (11). CAPM assumes that all investors tend to be risk-averse, which is not always true, as many investors choose to make risky investments for the chance of high returns. So, instead of CAPM, risk-adjusted return measures are needed for people who are more interested in higher-risk investments to determine the optimal investment portfolio. This study compares the accuracy of four risk-adjusted return measures: Sharpe ratio, Treynor ratio, Modigliani-Modigliani (M2) measure, and Jensen's alpha. All four of these measures are used to balance risk versus reward in possible investments (12).

The Sharpe ratio, named after its creator, William F. Sharpe, is a way to take risk into account while measuring the performance of an investment. It does this by comparing the investment to a risk-free asset, which is a US Treasury security in most cases and for this project (13). By using a risk-free asset as the benchmark, the measure focuses on the excess returns for each additional unit of risk taken on for the security they are investing in, breaking down risk and return to a simpler ratio (13). The Sharpe ratio was selected for this study because it is generally thought to be the most common risk-adjusted return measure currently used. The following is the formula used to calculate the Sharpe ratio:

$$Sharpe\ Ratio = (R_p - R_f) / \sigma_p$$

R_p is the expected return of the security, R_f is the risk-free rate, and σp is the standard deviation of the security (14). If the resulting Sharpe Ratio is negative, the risk-free rate is expected to be a better investment, or the investment could also be expected to depreciate based on historical trends. If it is between 0 and 1, it is typically considered a bad investment due to its amount of risk. 1-2 is regarded as a good investment, 2-3 is very good, and anything above 3 is excellent (14).

The Treynor ratio was created by Jack Treynor and was selected because it is like the Sharpe ratio, but with a different benchmark used to measure the performance. Whereas the Sharpe ratio measures the portfolio against a risk-free treasury security, the Treynor ratio uses beta to measure the portfolio against the market as a whole (15). Thus, the Treynor ratio is better suited for evaluating investments within a broader market context by taking into account market-wide risks (15). A stock's beta is the measure of systematic risk, or volatility, compared to the total market (16). A stock has a beta value above 1.0 if it swings more than the market, and a beta value below 1.0 if it swings less than the market (16). Swinging, in this case, refers to if a stock has a significant change in value over a short period of time (16). The more a stock swings, the more volatile it is, meaning that there is a higher risk but also a higher return potential. So, if the beta were equal to 1.0, that would mean the stocks have the same volatility as the total market (16). The following is the formula used to calculate the Treynor ratio:

$$Treynor\ Ratio = (R_p - R_f) / \beta_p$$

β_p is the beta of the portfolio and is calculated using the following formula:

$$\beta_p = Covariance\ (R_i, R_m) / Variance\ (R_m)$$

R_i is the average expected return of an asset, and R_m is the average expected return of the market. Treynor's ratio calculates the excess returns over the performance of the market (17). So if a portfolio has a 13% rate of return and the market has a 10% rate of return, only the extra 3% of the portfolio would be credited (17). However, if beta happens to be negative, the result from this measure will mean nothing (17). Otherwise, if a portfolio has a higher measure value, it means it's likely to be a good investment (17).

Another means to measure the risk-adjusted returns of an investment is the M2 measure. Its name originated from the father and granddaughter pair that created it: Franco and Leah Modigliani (18). Due to it being based on the Sharpe ratio, it is typically considered an extended and more useful version of its predecessor (18). Because it returns a percentage, comparing multiple portfolios is much simpler. The percentage tells the user how much risk is associated with an investment or portfolio, so comparing two securities becomes effortless. The following is the formula used to calculate the M2 measure:

$$M2\ measure = SR * \sigma_m + (R_f)$$

SR is the annual Sharpe Ratio of the investment, and σm is the standard deviation of the market. The M2 measure compares the returns of all portfolios if their risks were equal. So if there are multiple portfolios with different risk to return ratios, the M2 measure would scale them all to the same risk factor to then compare their risk-adjusted percentage of returns (19). For example, if one portfolio has a M2 value of 5.9% and another portfolio has a value of 5.4%, then the first portfolio would be 0.5% more profitable risk-adjusted performance (18). Thus, the M2 measure has greater applications in directly comparing two securities or portfolios.

Finally, Jensen's alpha, developed by Michael Jensen in 1968, was selected because it utilizes and expands on CAPM to calculate a portfolio's excess returns over its expected returns (20). Jensen's alpha focuses on the role of active management by determining whether a security is earning the proper amount of return for the amount of risk it has (20). In practice, this is crucial for evaluating the performance of mutual fund managers by evaluating the tradeoff of risk and return in the investment choices that the managers make (20). The following is the formula used to calculate the Jensen's alpha:

$$Jensen's\ alpha = R_i - (R_f + \beta_p * (R_m - R_f))$$

Jensen's alpha is simple to read: if the measure value is positive, then the investors earned enough to be compensated for the risk that they took with the investment in the first place. If the alpha is negative, then the investor did not earn enough to warrant the risk of the security in which they invested (21). Jensen's alpha compares the expected return of a security to the risk-adjusted, expected return of the market (21). Since it is risk-adjusted, if the security's expected return only marginally surpasses the risk-adjusted market's expected

https://doi.org/10.59720/24-016

return, the measure would return a negative value because the slight profits were not worth the risk (21). So, the security must beat the market by a significant enough margin such that the profits warrant the amount of risk taken.

The purpose of this study was to analyze the risk-adjusted return measures of the Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha, and determine which combination of measures would provide the most optimal portfolio to earn the greatest returns for the investor. We hypothesized that a combination of the four risk-adjusted return measures would more frequently predict funds with higher returns than the individual measures, with there being a better performance as more measures are used in tandem. However, our findings suggest that the combinations of measures actually degraded the overall performance of the portfolio, as the individual measures of Jensen's alpha and M2 measure performed better than all of the combinations. Investing can be risky if the investor does not have suitable knowledge of the market before entering, so this study could provide novice investors with a more focused investment strategy to gain high returns with lower risk.

RESULTS

To conduct this study, we retrieved monthly data from 1995–2020 for 20 mutual funds and calculated the measure values for the Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha to determine how profitable each fund was predicted to be by each measure. After retrieving all the funds' monthly data, the market returns of the fund and market, the standard deviation of the fund and market, the risk-free rate, beta, and the expected returns were calculated for each component.

Comparing how profitable each fund was predicted to be by each measure, we found that the funds Baron Partners Fund Retail Shares (BPTRX), Fidelity Growth Company Fund (FDGRX), and BNY Mellon Appreciation Fund, Inc. – Investor Shares (DGAGX) had very positive measure values for each of the four measures, which suggests that these three funds would be very profitable and a good investment (**Table 1**). Since the three listed funds are found in all four lists, they would average out to the same returns, but the funds that aren't common between all four lists would cause the differences in returns for each combination of risk-adjusted return measures.

However, the level of performance predicted by the risk-adjusted return measures did not always prove accurate with the actual return values (**Table 1 and 2**). Comparison between the percentage of returns calculated for each 5-, 10-, 15-, and 20- year period from 2000 and the risk-adjusted return measure values allowed us to see how accurate the predicted returns were for some funds. For BPTRX and DGRX, the measures proved extremely accurate, as after the first five years the two funds gave good positive returns, and after 20 years they were the highest performing funds with returns of 242.27% and 152.64%, respectively (**Table 2**). But in the case of DGAGX, the measures proved to be incorrect, as for every 5-year period DGAGX portrayed a trend of negative returns, indicating that the investor would lose money from having invested in the fund despite the high measure values (**Table 2**).

While building the portfolio for our study, we focused on the top five mutual funds, which were predicted to have the highest returns, from each measure because there was a need to limit the number of investments (**Table 3**). For each individual ratio, their portfolios consisted solely of the five predicted top-performing funds (**Figure 1**). For the combinations of risk-adjusted return measures, the top five lists were used once again. But since many of the funds overlapped, instead

Fund name	Sharpe	Treynor	M2	Jensen
DREVX	-0.93	-6.71	2.51	-18.59
CSIEX	0.86	5.71	9.12	-4.31
FFIDX	1.14	-5.62	9.11	-6.30
MIGFX	0.98	2.85	11.03	-7.67
DGAGX	3.72	13.71	18.43	2.58
ELFNX	2.06	9.92	12.64	-1.3
NICSX	0.64	-1.85	6.70	-10.69
EWMCX	-0.02	22.15	6.06	-11.56
AGRFX	1.26	-0.35	8.80	-5.86
BPTRX	2.01	13.70	13.28	0.6
PRWAX	0.66	1.17	6.38	-7.36
FDGRX	2.57	30.40	14.92	-1.17
EKOAX	1.37	-26.00	10.65	-6.53
APGAX	1.79	11.35	12.91	-0.77
OLGAX	1.31	2.98	9.54	-6.2
EPGAX	1.69	7.23	11.51	-2.67
ALARX	0.87	-8.28	8.87	-14.3
FDTOX	3.44	19.43	15.14	-6.12
AMAGX	2.78	-0.86	15.50	3.49
FAGAX	1.84	-1.82	9.79	-7.23

Table 1: Average annual risk-adjusted return measure values of each mutual fund from 1995-2000. The mutual funds were chosen due to having a lifespan from 1995 onwards and being large-cap funds that reduce risk by investing in reputable securities. The values were calculated by averaging the yearly risk-adjusted return measures of the Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha from 1995-2000 for all 20 individual mutual funds. The higher values generally indicated greater predicted returns relative to risk.

Fund name	2000 close values ($)	2005 returns (%)	2010 returns (%)	2015 returns (%)	2020 returns (%)
DREVX	11.20	-8.57	-21.98	-4.75	27.47
CSIEX	31.72	10.69	12.07	23.27	74.37
FFIDX	32.76	-2.91	-1.88	24.87	64.39
MIGFX	17.14	-28.88	-11.62	29.49	78.29
DGAGX	42.94	-7.72	-11.70	-7.19	-8.60
ELFNX	67.52	-28.57	-48.96	-21.26	5.80
NICSX	61.45	-5.43	-30.82	2.03	22.67
EWMCX	10.09	-18.09	-12.21	28.37	97.76
AGRFX	40.38	-6.89	-9.86	39.05	93.45
BPTRX	14.56	23.57	34.56	89.46	242.27
PRWAX	35.77	-11.51	-8.09	15.96	64.74
FDGRX	7.14	-11.56	15.19	65.08	152.64
EKOAX	27.49	-5.73	29.79	42.19	98.47
APGAX	26.71	-24.91	-7.54	34.81	102.78
OLGAX	20.13	-21.84	3.95	56.65	107.46
EPGAX	5.89	-20.42	-8.66	49.40	99.01
ALARX	15.48	-9.76	29.11	53.68	99.26
FDTOX	15.42	-9.45	-4.78	24.50	69.83
AMAGX	14.17	23.42	55.65	80.68	137.80
FAGAX	33.81	-3.61	0.97	56.84	143.42

Table 2: Market return of mutual funds for 5-, 10-, 15-, and 20-year periods starting in 2000. The market returns of each mutual fund assessed in this project. The mutual funds were chosen due to having a lifespan from 1995 onwards and being large-cap funds that reduce risk by investing in reputable securities. The 2000 close values were the closing values of each fund from December 31, 2000, which we used as the benchmark to calculate the percent returns of each 5-, 10-, 15-, and 20- year period.

https://doi.org/10.59720/24-016

Sharpe	Treynor	M2	Jensen
DGAGX	FDGRX	DGAGX	AMAGX
FDTOX	EWMCX	AMAGX	DGAGX
AMAGX	FDTOX	FDTOX	BPTRX
FDGRX	DGAGX	FDGRX	APGAX
ELFNX	BPTRX	BPTRX	FDGRX

Table 3: Top five mutual funds with highest predicted returns by each risk-adjusted return measure. Mutual funds were chosen based on the top five highest average annual measure values for each measure of Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha from 1995-2000.

of counting them multiple times, we still only counted them once, but then the funds that didn't overlap increased the total number of funds in our portfolio. For example, the combination of the Sharpe ratio and the M2 measure had four overlapping funds and one distinct fund each. So the new portfolio would consist of one of each of those four overlapping funds and both of the distinct funds, resulting in a total of six funds in the portfolio of that combination (**Figure 2**). The largest number of funds in a portfolio is in the combination of the Sharpe ratio, Treynor ratio, and Jensen's alpha, with a total of eight funds (**Figure 3**).

To find which combination of measures provides the most returns, we investigated the returns for each fund over four time intervals: 5, 10, 15, and 20 years. However, it is important to acknowledge that mutual funds are meant for long-term investments, and as such, the 5 and 10-year time periods did not show significant returns for many of the funds. Additionally, in the first decade, there were many outside factors that hurt the market, such as the bursting of the dotcom bubble in 2001 and the Great Recession in 2008. Therefore, the market returns worsened even further for those time periods. However, from 2010 onwards, larger returns and growth in the funds are seen because there were fewer negative outside influences.

For the 5-year time period from 2000-2005, the individual M2 measure had the most optimal portfolio return of 3.65%, and the individual Jensen's alpha held second place with 0.56% returns (**Table 4**). These low returns were to be expected because of both the long-term nature of mutual funds and the market crisis at the time.

For the 10-year time period from 2000-2010, the individual M2 measure maintained its position as the most optimal portfolio with 17.78% returns, and the individual Jensen's alpha stayed close behind with 17.23% returns (**Table 4**). Here, we saw a slow rise in the profitability of the mutual funds, but there is still much space to grow.

For the 15-year time period from 2000-2015, the individual Jensen's alpha took over as the most optimal portfolio with 52.57% returns, and the individual M2 measure fell to second with 50.50% returns (**Table 4**). Here, we saw a higher rise in returns of the funds as the market stayed relatively steady with no major destabilizing events during that time period.

For the 20-year time period from 2000-2020, the individual Jensen's alpha was the most optimal portfolio with 125.38% returns, whereas the individual M2 measure maintained second with 118.79% returns (**Table 4**). Since mutual funds are meant to be long-term investments, it was expected that the highest returns would be after the longest period of time. Overall, our data suggest that the individual M2 measure provides the best returns to construct the most optimal portfolio within 5 and 10 years, while the individual Jensen's alpha gains the investor the best returns over 15 to 20 years. As for the combinations, the difference between the top performing measure, Jensen's alpha, and our hypothesized combination of all four measures is 25.35%, a very large decrease in profits because of the usage of all the measures.

DISCUSSION

In this study, we analyzed the risk-adjusted return measures of the Sharpe ratio, Treynor ratio, M2 measure, and Jensen's alpha to determine which combination of measures would provide the most optimal mutual fund investment strategy. We hypothesized that as more measures were used

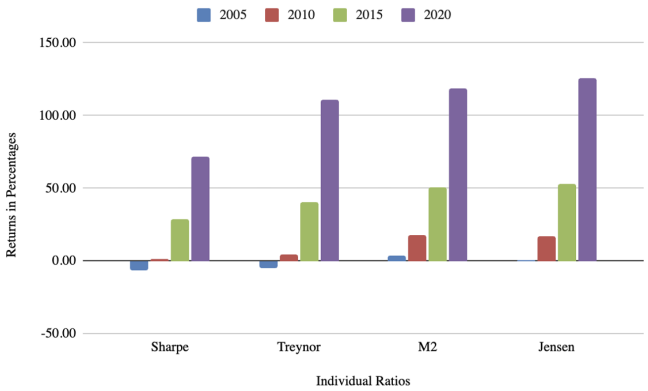

Figure 1: Market returns for each individual risk-adjusted return measure's mutual fund portfolio for each 5-year increment. Market returns of the individual measures' portfolios for each 5- year (blue), 10- year (red), 15- year (green), and 20- year (purple) period from 2000. The returns are expressed as percentages and highlight the performance trends of each measure's selected portfolios over time. The portfolios were chosen based on the measure's predicted top five performing mutual funds.

Figure 2: Market returns for each combination of two risk-adjusted return measures' mutual fund portfolio for each 5-year increment. Market returns of combinations of two measures for each 5- year (blue), 10- year (red), 15- year (green), and 20- year (purple) period from 2000. The returns are expressed as percentages and highlight the performance trends of each combination's selected portfolios over time. The portfolios were chosen based on the measures' top five list, including all distinct mutual funds once, even if they showed up in both lists. Additionally, the total number of funds in each combination's portfolio is shown in parentheses with the names.

https://doi.org/10.59720/24-016

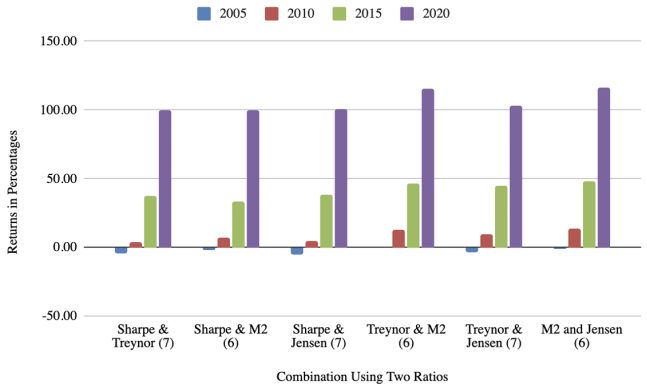

Figure 3: Market returns for each combination of three risk-adjusted return measures' mutual fund portfolio for each 5-year increment. Market returns of combinations of three measures for each 5- year (blue), 10- year (red), 15- year (green), and 20- year (purple) period from 2000. The returns are expressed as percentages and highlight the performance trends of each combination's selected portfolios over time. The portfolios were chosen based on the measures' top five list, including all distinct mutual funds once, even if they showed up in multiple lists. Additionally, the total number of funds in each combination's portfolio is shown in parentheses with the names.

Combination	# of funds	2005 (%)	2010 (%)	2015 (%)	2020 (%)
Sharpe	5	-6.78	1.08	28.36	71.49
Treynor	5	-4.65	4.21	40.04	110.78
M2	5	3.65	17.78	50.50	118.79
Jensen	5	0.56	17.23	52.57	125.38
Sharpe &Treynor	7	-4.06	3.96	37.09	99.64
Sharpe & M2	6	-1.72	6.66	33.67	99.96
Sharpe & Jensen	7	-5.03	4.63	38.01	100.36
Treynor & M2	6	0.03	12.78	46.82	115.28
Treynor & Jensen	7	-3.53	9.88	45.10	103.52
M2 & Jensen	6	-1.11	13.56	47.89	116.12
Sharpe, Treynor, & M2	7	-4.06	3.96	37.09	99.64
Sharpe, Treynor, & Jensen	8	-6.66	2.53	36.81	100.03
Sharpe, M2, & Jensen	7	-5.03	4.63	33.83	100.36
Treynor, M2, & Jensen	7	-3.53	9.88	45.10	103.52
Sharpe, Treynor, M2, & Jensen	8	-6.66	2.53	36.81	100.03

Table 4: Market returns for each combination of risk-adjusted return measures for 5-, 10-, 15-, and 20- year periods starting in 2000. The top five funds from each measure were used to create portfolios and calculate the returns of each portfolio of mutual funds for the 5-, 10-, 15-, and 20- year periods. Combinations with different top five funds had portfolios including all distinct mutual funds once, even if they showed up in both lists, resulting in a larger total number of funds.

in conjunction, the portfolio would perform better because each individual measures already attempts to find the best mutual funds to use for the investor. However, we discovered that some of the individual measures performed better than the combinations, namely, the individual Jensen's alpha and M2 measure. Although it cannot be said for certain why the individual measures performed better, it is likely due to each measure having different methods of calculating the best funds to invest in, which, when used in conjunction, may overlap with or contradict each other.

To put it into perspective the vast difference between the individual Jensen's alpha and the combination of all four measures, if $50,000 was invested 20 years ago using funds solely chosen by Jensen's Alpha, the investor would now have a balance of $112,690. On the other hand, if the combination of all four measures was used, then the balance would be $100,015. With over a $12,000 difference, or 25.35%, the measure that an investor chooses to follow matters greatly.
While examining our study, an important note to consider is that the investment method we used was a lump sum investment in December of 2000. We did not consider additional investments or withdrawals throughout the 20-year period that we studied. A lump sum investment is not how people usually invest in real life as they would be slowly adding or changing investments. Therefore, profits may not have been maximized, but additional investments would have been subjective to the investor and cannot be calculated by a study, hence a common way to investigate rates of return on investments is through a lump sum investment.

Furthermore, external factors caused a bearish market, where stock prices are expected to go down, for nearly a decade during the study period, resulting in very low returns for that period of time. Although it would be beneficial to maintain a neutral or bullish market, where stock prices are expected to rise, to maximize profits, by involving external factors, this study maintains a sense of realism because investors cannot anticipate everything that happens to the market.

This study could be expanded by using a larger sample size or a variety of different types of mutual funds. In this study, we only used large growth mutual funds, but incorporating a variety such as small and medium growth funds would help account for all sorts of funds in the results. Additionally, the time periods could be increased from 20 years to even longer. Considering that mutual funds are commonly used for retirement funds such as a 401k, a longer period of about 40 years would more accurately represent the returns that an investor would earn on their retirement balance.

Finally, it is important to note that the longer time periods matter most because mutual funds tend to be used more often for long-term investments, such as retirement accounts (6). Thus, the most lucrative measures were the individual Jensen's alpha and the individual M2 measure. However, the number of mutual funds used for each combination must also be considered. For a more risk averse investor, a higher amount of mutual funds would be beneficial because it spreads the risk so that even if one fund fails, the others would still have high returns. Whereas if there are only a few mutual funds, if even one fund fails, it would have a large negative impact on the overall portfolio returns. So, although the individual Jensen and M2 measures provide the most returns, when compared to other combinations, some investors may choose to invest in a combination with a larger number of mutual funds as well as high returns to be able to offset the negative impact of a possible failed fund.

MATERIALS AND METHODS

For this study, we identified 20 mutual funds with the criteria for selection requiring the funds to be large growth funds, and to have data from on or before 1995. Then we retrieved the monthly historical data for all 20 funds, the monthly historical data for the market, or benchmark, which is the Dow Jones Industrial Average (DJIA), and the monthly historical data for the Treasury Yield 10 Years (^TNX). The Treasury Yields are government issued securities with guaranteed returns, thus they are used as the risk-free rate.

All data calculations were conducted from 1995 to 2020.

https://doi.org/10.59720/24-016

Firstly, we calculated the monthly returns of the each individual fund and the market by using the formula of ln(m2/m1)*100, where m2 is the normal close value of the current month, and m1 is the normal close value of the previous month. After the monthly returns, we repurposed the formula to find the yearly returns of each individual fund, the market, and the Treasury Yield using ln(y2/y1)*100, where y2 is the normal close value of December of the current year, and y1 is the normal close value of December of the previous year.

Next, we calculated the beta of each mutual fund using the covariance-variance method: β_p = Covariance (Fm1:Fm12, Mm1: Mm12) / Variance (Mm1: Mm12), where Fm1:Fm12 are all the monthly returns of the funds for one year, and Mm1:Mm12 are all the monthly returns of the market for one year. After calculating beta, we had all the parts necessary to find the expected returns of the fund by using the formula of $R_f + \beta^*(R_m - R_f)$, where Rf is the risk-free rate, β is beta, and R_m is the yearly return of the market. Finally, we also calculated the standard deviation of the monthly returns of both the market and each individual mutual fund. With these, all the calculations were ready to move on to the actual risk-adjusted return measures.

For each fund, the measures were calculated in the order of Sharpe ratio, Treynor ratio, Modigliani-Modigliani measure, and finally, Jensen's alpha by using their respective formulas.

$$\text{Sharpe ratio} = (R_p - R_f) / \sigma_p$$
$$\text{Treynor ratio} = (R_p - R_f) / \beta_p$$
$$\text{M2 measure} = SR^* * \sigma_m + (R_f)$$
$$\text{Jensen's alpha} = R_i - (R_f + \beta_p^*(R_m - R_f))$$

R_p is the expected return of the fund, R_f is the risk-free rate, σ_p is the standard deviation of the fund, β_p is the beta of the portfolio, SR is the annual Sharpe ratio of the investment, and σm is the standard deviation of the market.

The time period of 1995-2000 was used as the starting point to find each measure's predicted mutual funds, while the time periods after were used to determine the effectiveness of each measure. So, we made a table of values for each fund by using the averages of each measure value from 1995-2000, which would serve as the prediction of the funds' performances. Then, we made a table that used the historical data from 2000-2020 to calculate the actual returns of each fund for 5-, 10-, 15-, and 20- year time periods. Next, we made a top five list for each risk-adjusted return measure which included their predicted top performing funds. Finally, we created the portfolios for each individual measure and for each combination of measures by using the top five lists. Since the combinations included the top five lists from multiple measures, their portfolios included all distinct mutual funds, meaning that if a fund appeared multiple times, it would still only get added to the portfolio once. But each unique mutual funds would get added to the portfolio, sometimes resulting in more than five funds in the portfolios of the combinations. We analyzed these portfolios by comparing them to actual return values of the funds over the 20 years to determine the accuracy of the funds and whether they worked better individually or in combination.

Received: January 2, 2024
Accepted: June 16, 2024
Published: May 5, 2025

REFERENCES

1. "Total Market Value of the U.S. Stock Market." *Siblis Research*, 13 Sept. 2024, siblisresearch.com/data/us-stock-market-value/. Accessed 15 Oct. 2024.
2. Singletary, Michelle. "Advice | More Americans Own Stocks. This Is Great for Their Financial Future." *The Washington Post*, WP Company, 9 June 2023. http://www.washingtonpost.com/business/2023/06/06/american-stock-ownership/. Accessed 29 Oct. 2023.
3. Holden, Sarah, and Daniel Schrass. "Mutual Funds: Rated G-All Audiences Admitted." *Investment Company Institute*, 4 Feb. 2021.
4. "Understanding Stocks." *Schwab Brokerage*, www.schwab.com/stocks/understand-stocks. Accessed 15 Oct. 2024.
5. "Understanding Mutual Funds." *Schwab Brokerage*, 16 Jan. 2021. www.schwab.com/mutual-funds/understand-mutual-funds. Accessed 24 Apr. 2024.
6. Iachini, Michael. "Are Mutual Funds a Good Investment?" *Schwab Brokerage*, 18 Dec. 2022. www.schwab.com/learn/story/are-mutual-funds-good-investment. Accessed 14 Oct. 2024.
7. Yochim, Dayana. "Index Funds vs. Mutual Funds: The Differences That Matter." *NerdWallet*, 14 July 2023. www.nerdwallet.com/article/investing/index-funds-vs-mutual-funds. Accessed 24 Apr. 2024.
8. "Index Funds vs. Actively Managed Funds." *Vanguard*, 19 Feb. 2022. investor.vanguard.com/investor-resources-education/understanding-investment-types/index-funds-vs-actively-managed-funds. Accessed 24 Apr. 2024.
9. Sahu, Sridhar. "What Is Large Cap Equity Mutual Fund?" *ET Money*, 19 Mar. 2024. www.etmoney.com/learn/mutual-funds/large-cap-mutual-funds/. Accessed 24 Apr. 2024.
10. Chen, James. "Blue Chip Meaning and Examples." *Investopedia*, 9 July 2024, www.investopedia.com/terms/b/bluechip.asp. Accessed 15 Oct. 2024.
11. Kenton, Will. "Capital Asset Pricing Model (CAPM) and Assumptions Explained." *Investopedia*, 24 May 2023. www.investopedia.com/terms/c/capm.asp. Accessed 14 Oct. 2024.
12. "Understanding High-Risk Investments." *FCA*, Financial Conduct Authority, 13 Oct. 2023. www.fca.org.uk/investsmart/understanding-high-risk-investments. Accessed 14 Oct. 2024.
13. Fernando, Jason. "Sharpe Ratio: Definition, Formula, and Examples." *Investopedia*, 4 Aug. 2024, www.investopedia.com/terms/s/sharperatio.asp. Accessed 15 Oct. 2024.
14. Baldridge, Rebecca, and Benjamin Curry. "Understanding the Sharpe Ratio." *Forbes*, Forbes Magazine, 14 Dec. 2022. www.forbes.com/advisor/investing/sharpe-ratio/. Accessed 14 Oct. 2024.
15. Maverick, J.B. "Sharpe Ratio vs. Treynor Ratio: What's the Difference?" *Investopedia*, 31 Aug. 2024, www.investopedia.com/ask/answers/010815/what-difference-between-sharpe-ratio-and-traynor-ratio.asp. Accessed 15 Oct. 2024.
16. Liberto, Daniel. "What Beta Means When Considering a Stock's Risk." *Investopedia*, 12 July 2023. www.investopedia.com/investing/beta-know-risk/. Accessed 14 Oct. 2024.

https://doi.org/10.59720/24-016

17. "Treynor Ratio." *Wall Street Prep*, 30 Aug. 2023. www.wallstreetprep.com/knowledge/treynor-ratio/. Accessed 14 Oct. 2024.
18. Srivastav, Ashish Kumar. "M2 Measure (Definition, Formula) | Examples to ... - Wallstreetmojo." *Wall Street Mojo*, 13 Sept. 2022. www.wallstreetmojo.com/m2-measure/. Accessed 14 Oct. 2024.
19. Thakur, Madhuri. "M2 Measure: Advantages and Disadvantages of M2 Measure." *EDUCBA*, 17 July 2023. www.educba.com/m2-measure/. Accessed 14 Oct. 2024
20. Raichura, Sandip. "What Is the Jensen's Alpha?" *PLblog*, 19 May 2018. www.plindia.com/blog/what-is-the-jensens-alpha/. Accessed 14 Oct. 2024.
21. Chen, James. "What Is Jensen's Measure (Alpha), and How Is It Calculated?" *Investopedia*, 30 Sept. 2020. Accessed 14 Oct. 2024.