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Article

Effects of noise on information corruption in the quantum teleportation algorithm

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SUMMARY

In the quickly growing field of quantum computing, interference called "noise" causes discrepancies within the results of a quantum experiment. We aimed to observe the effects of noise on the quantum teleportation algorithm, which is used to transport the quantum state of one qubit to another qubit over distances. When the information in the teleportation algorithm is passed through noisy channels, the data can easily become corrupted. We conducted this study in order to observe the effects of noise on the information that travels through these channels. We aimed to understand the nature of the data corruption to help create improved and more faithful methods of data communication. Prior to conducting this study, we expected that the noise would have the most effect on the real and simulated backends and the least effect on the perfect backend. Through this study, we ran teleportation circuits on three different backends (a perfect simulation, a noisy simulation, and the real hardware) and compared the final results with our expected states. We observed that the predicted effects of noise on the information is accurate in some respects but that noise has the potential to drastically alter and corrupt the data. Moreover, the real hardware we used was far more susceptible to noise interference than the simulations. Our findings indicate that, to prevent the information from being compromised, other methods of noise reduction must be employed in conjunction with the quantum teleportation algorithm.

INTRODUCTION

Quantum computing, the form of computing that makes use of qubits – basic units in quantum computing, similar to classical bits but able to exist in a state between 0 and 1 – to perform large and complex calculations, is widely viewed as a burgeoning field of more powerful computing (1,2). With the increase in power, there are many uses for quantum computing. One of these uses is the accurate and secure transfer of information (1). Picture a scenario where two people, Alice and Bob, wish to exchange information stored in a qubit. To send the information, Alice must store the qubit's information in two classical bits, which are then sent to Bob, who will read the information and store it in his qubit. In the process of transferring her qubit's information, Alice must use a series of operations – otherwise referred to as "gates" – to perform an entanglement, which connects qubits so that each qubit's state is reliant on the other (1-4).

The quantum teleportation algorithm relies on a series of entanglement operations and measurements which collapse the original qubit state and re-form it on the target qubit. However, a common occurrence in quantum computing is noise which is an unwanted interaction with the environment that may alter a qubit's state (2). Noise can drastically affect the gate fidelity – a measure of how secure a qubit operation is – and therefore can compromise the information being transmitted (3,5). It has been observed that the type of noise the circuit is subjected to can dramatically change the extent to which the information is changed (4). Other papers have examined the effects of bit-flip, phase-flip, amplitude dampening, and depolarization errors on gate fidelity, finding that the errors led to the degradation and occasional collapse of qubit states (2).

Researching the effects of noise on the quantum teleportation circuit and how noise affects the information transfer is important because it will help quantum computers understand noise to a greater degree. It can also help mitigate the issues created by noise. Researching the issue of noise within quantum computers will enable researchers to have a better grasp on how information can be changed or corrupted when sent through noise channels (4).

We aimed to explore and observe the effects that noise has on the information transfer through a quantum teleportation algorithm. We observed the information transfer by running the quantum teleportation circuit through three different backends (quantum computers on which the circuits are run) – one perfect backend, one real backend, and one simulated backend – and comparing the sent and received qubit states. We expected that the real and simulated backends would show the greatest discrepancies between the sent and received states, and that the perfect backend would show the least discrepancies. Our findings closely adhered to these expectations.

RESULTS

Our study aimed to compare the sent and received states of the qubit that goes through the teleportation algorithm. To measure how accurately data is preserved, we sought to observe any differences between the expected result and the actual result, as any interference from noise would manifest itself in the received state. For the portion of this study that uses histograms to visualize data, we found that the circuit needed to be measured twice: once for the sent state and once for the received state. However, we could not measure the qubit containing the sent state without collapsing the quantum state, creating the need for two circuits (sequences of quantum gates), one to measure the sent state (**Figure 1a**) and one to measure the received state (**Figure 1b**).

During this study, we ran each circuit on each backend

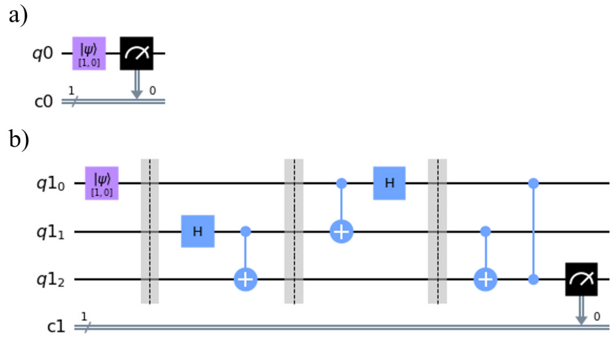

Figure 1: Circuits measuring sent and received states. (a) A qubit, initialized in the [1, 0] state, is immediately measured and stored in the classical register. (b) Three qubits are connected through entanglement, allowing the state of the first (q10) qubit (initialized as [1, 0]) to be transferred to the second (q12) qubit, after which the second qubit is measured and stored in the classical register.

twice: once to perform a Quantum State Tomography and once to get the counts for a histogram. Counts are a measure of the results of quantum circuits; when the qubits are measured, they take either of the classical states, 0 or 1. In this study, "counts" refers to the results of the different trials. We ran the circuits on a series of backends: the perfect backend (with no noise), the real backend (with real noise levels), and the backend simulating the real backend (with added noise to mimic real noise levels). The purpose of using multiple backends was to assess any effect the backend had on the noise levels.

Quantum State Tomography (QST) is a method used to analyze the results of a quantum experiment to reconstruct the original qubit states (2). The QST part of the research involved creating a QST experiment for each backend and retrieving the resulting density matrix. We visualized the matrix in two different ways within this study. The first visualization is a cityscape model that displays the probabilities for real and imaginary states obtained during the experiment (**Figure 3a-c**). For the perfect simulation, the pure and mixed states with the highest probability all measured the expected zero state for qubit 2 (**Figure 3a**). The results from the noise model had a similar but diminished probability for the group and instead showed an increased probability for the pure states (**Figure 3b**). The real hardware largely followed the trend, and preferred mainly pure states (**Figure 3c**).

The second visualization used in this study that makes use of the QST experiments is the Bloch sphere model (**Figure 4a-c**). A Bloch sphere model is a representation of a quantum state vector (3, 4). As qubits adopt a value between 0 and 1, the vector representation allowed us to observe the superposition of the states and ascertain the composition of a quantum state. Using the density matrices, we found it possible to re-create the received quantum state. In the circuit measuring the received state (**Figure 1b**), we teleported the state from the first qubit (qubit 0) to the third qubit (qubit 2). Thus, after a successful run, the state visualized for the third qubit in each set should have matched the sent state (**Figure 2**).

Aside from the QST experiments, we ran the circuits (**Figure 1a-b**) again on the backends to get the results for a histogram (**Figure 5**). The results were the values – either 0 or 1 – obtained from measuring the qubits containing the sent and received states. As shown in the histogram (**Figure 5**), the perfect simulation consistently measured the same state as was sent, with no errors. We followed the simulation with the noise model, for which there existed a 0.031 difference between the probabilities. The most 'inaccurate' was the real hardware, which had a 0.077 difference in probability, more than two times that of the noise model.

By using the same counts shown in the histogram, we determined the Hellinger fidelity, which is equivalent to the quantum state fidelity (i.e., a measure of how close two quantum states are to each other). The Hellinger fidelity is defined as $(1 - H^2)^2$, with H representing the Hellinger distance (6) which is the difference between the probability distribution of the counts (7). The closer each fidelity is to 1 (the value calculated when the difference in probability, H, is 0), the closer the resulting state is to the initial sent state (6). Similarly, the farther each fidelity is from 1, the more the data is corrupted from the influence of noise. The Hellinger fidelity for the perfect simulation was 1.0, the fidelity for the noisy simulation was 0.994, and the fidelity for the real hardware was 0.963. Looking solely at the Hellinger fidelities, it is evident that the perfect simulation was, in fact, perfect. Between the noisy simulation and the real hardware, there was a difference of 0.031.

However, as demonstrated through the Bloch spheres (**Figure 4a-c**), all three re-created state vectors were fairly accurate. When each third Bloch sphere is compared to the sent state (**Figure 2**), each backend produces some level of accuracy, though not all of them are equal. While the perfect simulation is the closest to the sent state, the real hardware is the furthest, being slightly off the pole and having a diminished magnitude. As shown through the perfect simulation, the expected received state vector should have been a vector of magnitude 1, pointing directly upwards – otherwise known as being aligned with the pole. The inconsistencies in the attributes of the state vector show the inaccuracy of the real hardware.

Whereas the Bloch spheres show how each backend is similar and accurate, the cityscape model shows how different the results are from the three backends. The results of the QST experiment on the perfect simulation (**Figure 3a**) look far

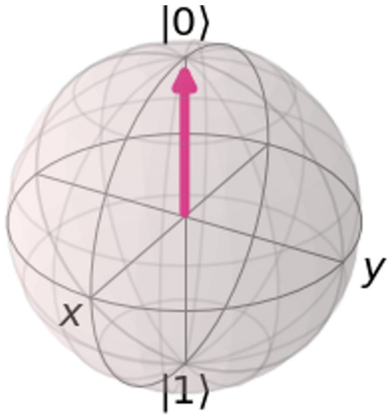

Figure 2: Bloch sphere visualization of the sent state. Bloch sphere showing the sent state [1, 0].

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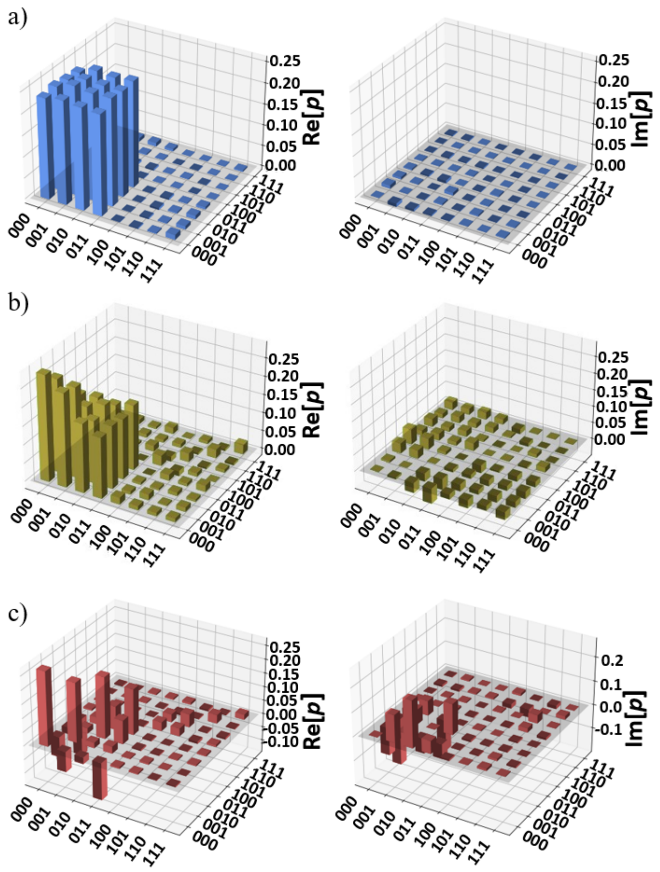

Figure 3: Cityscape models visualizing the density matrix of real and imaginary states measured from the different backends. QST experiment was conducted measuring the results of the teleportation circuit on the (a) perfect backend, (b) noisy simulation, and (c) real backend.

more uniform than the results of the noise model (**Figure 3b**), which are still orderly to some degree. The most unorderly results, however, are those of the real hardware (**Figure 3c**). Although the Hellinger fidelity of the real hardware shows that it is fairly accurate, there is no visible pattern in the cityscape model for the backend (at least, not like the patterns for the perfect and noise simulations (**Figure 3a-b**).

DISCUSSION

We expected that the perfect simulation would be the most "accurate" (i.e., how closely the expected state aligned with the actual state), followed by the noisy simulation and the real hardware. The noisy simulation and the real hardware should have been closely related since one is a simulated version of the other. The results closely resembled our expectations.

Combining the information from each model, it became evident that the real hardware is somewhat close to the accuracy of the perfect simulation, though the information is still altered due to the noise channels. In addition, though each backend's measured state vector (as shown through the Bloch sphere visualization) was similar to the expected state vector, the direction and magnitude of the real hardware's received state vector were different. This is likely because of the mixed state it represented. Also, the perfect simulation and the noise model were slightly more uniform in their density matrices, while the real hardware was more random. Moreover, the Hellinger fidelities helped in ranking the accuracy of each model: the closer the fidelity was to 1, the more the received state aligned with the expected state. By each test, the order of the backends, from most accurate to least accurate, was the perfect simulation, followed by the noisy simulation, followed by the real hardware. These findings aligned with our initial hypothesis.

In general, some aspects of the results were expected, such as the fidelities and the differences between the cityscape plots of the QST. Unexpectedly, the noise model functioned, to a certain extent, better than the backend it represented.

There were limitations with the procedure of this study. For one, the reconstructed states from the QST experiments naturally deviate from the original states; QST experiments make use of measurements from a finite number of runs, yielding a close, but not exact, approximation of the quantum state (8). Some papers have investigated the use of compressed sensing (reconstructing a state vector to a limited degree with a smaller number of measurements) to address the discrepancy (9). Another issue arose from the limitations of modern quantum hardware. As it currently stands, conditional gates are not supported on the available quantum computers, since measuring a quantum state causes it to collapse into a classical state. To gain the ability to run the circuit on the backends available, we modified the circuit so that it utilized controlled X and Z gates as opposed

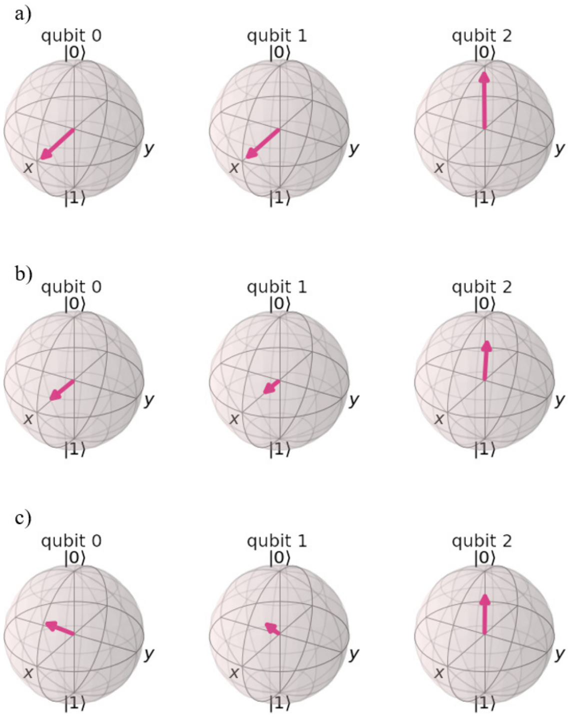

Figure 4: Bloch spheres showing the reconstructed states of the three qubits at the end of the teleportation circuit run on the different backends. QST experiment measured results of the teleportation circuit and reconstructed states were visualized for the (a) perfect backend, (b) noisy backend, and (c) real backend.

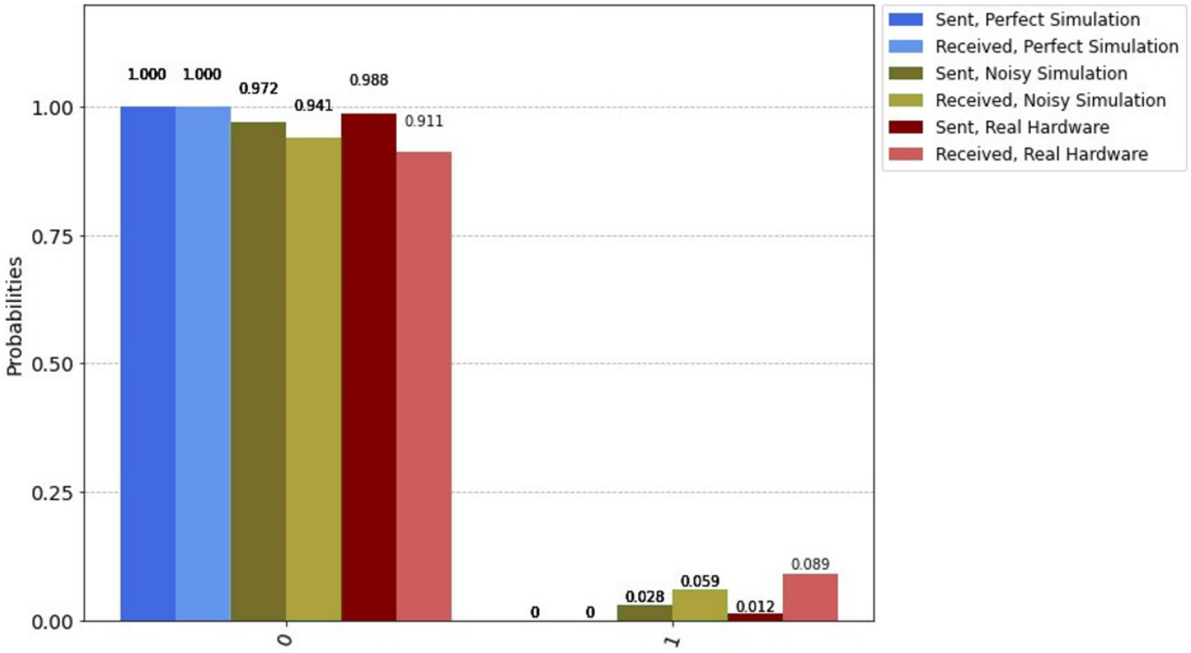

Figure 5: Counts measured from quantum teleportation circuit on different backends. Comparison between sent and received qubit states through the teleportation circuit. Probabilities measured from both ends of communication from each backend. Two circuits, measuring the sent and received states, were executed with 1024 shots on each backend. The probabilities listed (ranging from 0 to 1) show the percentage of shots that yielded a classical state (either 0 or 1).

to conditional X and Z gates. As a result, the information was not stored on the classical registers, though the circuit's function was not affected.

QST has been observed for the ability to reconstruct quantum states in the field of cryptography and for the ability to characterize the movement of atoms and trapped ions in other studies (9,10). This paper aimed to make use of QST to shed light on the nature of quantum computing itself, with a focus on the effects of noise.

As quantum computing grows to be used for more applications, it is important to study and address its potential flaws. Through this study, we determined that the noise experienced on real hardware makes calculations imperfect, and that the noisy simulation was more accurate than the backend it modeled. Future study could potentially investigate methods of eliminating /suppressing interference or creating more accurate simulations.

MATERIALS AND METHODS

In this study, IBM Quito was used as the real hardware backend and is available for public use through IBM's Quantum Services. In addition to the real hardware, the Qiskit Aer simulator (version 0.13.0) was used to simulate an ideal, noise-free environment. A Quito noise model, provided by Qiskit (version 0.45.1) was also used. Qiskit, an open-source software development kit based in Python (version 3.7.17), includes commands which were used during this study to conduct QST experiments, calculate Hellinger fidelities, and visualize Bloch spheres.

As mentioned, two circuits were constructed and used to allow for the measuring of both the sent and received states (**Appendix**). The first circuit (**Figure 1a**) contained a single qubit and measurement. The second circuit (**Figure 1b**) contained the regular teleportation circuit with a measurement at the end. To collect the results for the QST experiments and the histogram, each circuit was run with the specification of 1024 shots, meaning each circuit was run 1024 times. The reconstructed quantum states were the averages of the results from each execution of the circuits.

In this study, a state vector in the [1, 0] state was used as the 'sent' state.

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