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Article

A new scale of mathematical problem complexity and its application to understanding fear of mathematics

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SUMMARY

Fear of mathematics appears to be a global phenomenon. However, it has primarily been studied in isolation despite mathematics being part of a school curriculum with multiple subjects. We reason that the relative difficulty of mathematics compared to other subjects could be a factor behind the fear of mathematics. We explored this line of reasoning by hypothesizing that middle and high school mathematics is more complex than English literature. To enable the testing of this hypothesis, we developed scoring methods for complexity. While the English complexity score was derived from the Flesch reading ease score, we introduced a new way of measuring the complexity of mathematical problems. Using these metrics on samples for English prose and mathematical problems for grades 5 through 11, we then established that mathematical complexity is relatively high. Analysis of intergrade changes in scores showed significant changes in mathematical complexity between grades 5 and 6, and grades 6 and 7. We recommend that mathematics curricula be spread over a more extended period or that the time spent on mathematics in school be increased to address this potential factor behind the fear of mathematics.

INTRODUCTION

Fear of mathematics appears to be a global phenomenon. A study in the United States found that only 11 of the 157 participants had positive experiences with mathematics from kindergarten through college (1).

Several studies worldwide (United Kingdom, New Zealand, Australia, Germany, Brazil, Nigeria, Pakistan, and Nepal) have reported fear of or anxiety towards mathematics (2–9). For example, a survey of 1,600 students in India found that 82% of students in grades 7 through 10 feared mathematics, which increased with grade level (9).

Researchers have proposed several theories about the cause of this fear. A well-cited theory is that students pick up their fear in multiple ways: from their parents at home, their teachers in class, and society at large (3). This theory suggests that parents could unintentionally transfer mathematical anxiety to their children. Teachers who accepted and promoted only one ‘right answer’ could also cause mathematics anxiety in students. Many myths about mathematics, such as that only people born with a ‘math mind’ can understand mathematics, could add to students' fears.

Another line of research attributes mathematics anxiety to the boring nature of teaching mathematics. It has been recommended to increase the use of pictures in primary grades to help reduce anxiety in students (4). Inability to solve problems in time, aggression by teachers, or dyscalculia (mathematical learning disorder) could be some causes of the fear of mathematics (8).

Moreover, the recently published National Curriculum Framework for School Education in India discussed many essential points about the fear of mathematics and steps that should be taken to prevent it (10). It highlighted that if the basics of numerical literacy were unclear, the students would not understand more advanced and essential topics. It suggested that there were two major factors behind the fear of mathematics. The first was the teaching method, and the other was societal perceptions. The solution proposed was for the teachers to use teaching aids to help students lagging in understanding. Additionally, they suggested that the students should not be limited to only one answer and should be allowed to explore multiple strategies to reach an answer.

As seen from the above overview, fear of mathematics has been studied in isolation, looking at mathematics as a stand-alone subject. However, mathematics is part of a curriculum that includes languages, natural sciences, and social sciences. If mathematics were harder compared to the other subjects, it would demand that students spend more time on it. Yet, it is a natural tendency to gravitate to topics that are easier to grasp (11). Therefore, time spent on mathematics would decrease. These two factors will keep reinforcing each other. Evidence supports that the performance level in mathematics is positively correlated with the time spent on mathematics (12). Thus, the progressive widening of the gap between the demand and supply of time on mathematics will lead to a progressive decline in performance. This downward spiral could eventually lead to fear of mathematics. Therefore, we reasoned that the relative complexity of the mathematics curriculum, when compared to other subjects, could be a factor behind the fear of mathematics. We explored this line of reasoning by hypothesizing that the complexity of mathematics is higher than that of English literature in the school curriculum.

To test this hypothesis, we needed to develop scoring methods for measuring complexity. The Flesch reading ease score has been used for decades to assess the complexity of texts and the reading level required of the reader (13). This was amenable to be converted to a complexity score. Also, researchers have studied mathematical complexity in multiple contexts. For example, a processing model explicitly deals with the text comprehension and problem-solving

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aspects of word arithmetic problems (14). There are reports of ways to quantify cognitive complexity levels in mathematical problem-solving items (15). Methods exist to differentiate between micro- and macro-levels of mathematical complexity based on two primary ways teachers use their content knowledge: decompressing and trimming (16). While all these methods have their utility, we needed a scale of mathematical complexity that considered the aspect of conceptual difficulty.

Conceptual difficulty relates to the pre-requisite level of knowledge required to understand and solve the problem. For example, to solve '2+3', knowledge of the addition operation is needed. However, to solve '2-3', knowledge of the subtraction operation is required. By merely practicing additions, a student will not be able to solve subtraction problems; a new concept will need to be taught. Also, in order of concept introduction, addition must precede subtraction. Textbooks follow the same logic in sequentially introducing the concepts as the students move up the grades.

Now, even if two problems have the same level of conceptual difficulty, they may differ in their computing difficulty depending on the number of steps required to arrive at a solution—the more the number of steps, the greater the computing difficulty. The computing difficulty cannot be taken as linearly proportional to the number of steps because the contribution to computing difficulty, though positive, tapers for every new step added. To illustrate, consider two problems of the same conceptual difficulty that differ only by one solution step. Suppose one of them takes 10 steps and the other 11. The last step can be taken as adding a 10% (1/10) burden. However, if the problems took 20 and 21 steps, respectively, the last step would add only a 5% (1/20) burden.

Keeping the above considerations in mind, we developed the scoring methods appropriate for this study. Using these metrics on samples for English prose and mathematical problems for grades 5 through 11, we then established that mathematical complexity is relatively high. Thus, while the fear of mathematics is something a student experiences inside, this study suggests that one of the causes may lie outside in how the curricula are structured and how much time is devoted to mathematics.

RESULTS

In order to compare the complexities of mathematics and English, we developed a scoring method. Briefly, this method scores the mathematical complexity of a problem as the product of the conceptual difficulty (indicated by the order of introduction of the concept related to the problem in the curriculum) and the computing difficulty (indicated by the number of steps required to solve the problem). The English reading complexity is a score derived from the widely used Flesch reading ease score. Problems sampled from the textbooks for grades 5 through 11 were scored by hand (mathematics) or using software (English) and normalized for comparison.

The normalized complexity scores increased linearly with grades (English, R^2 = 0.884; mathematics, R^2 = 0.966). This was in line with the expectation that complexity should increase with grade level. The normalized complexity score in mathematics was higher than that of English. Between grade 5 and grade 11, the complexity score of English increased 1.7-fold (from 5.0 to 8.3), while mathematics increased 3.6-fold (from 5.0 to 18.1). The complexity score of mathematics was significantly higher than that of English (p = 0.012, **Figure 1**).

Moreover, the changes in complexity scores from grade to grade were not uniform. Even as English complexity scores did not change much, mathematics normalized scores doubled between grades 5 and 7 (**Figure 1**). Intergrade average score changes were not statistically significant, except for mathematics between grades 5 and 6 (p = 0.0014), and grades 6 and 7 (p = 0.0001), with the latter showing the highest change (**Figure 2**).

An additional outcome of our study was deriving the English complexity scores from the Flesch reading ease score. There was a negative relationship between reading ease score estimates (interpolated from the suggested ranges) against the suggested grade levels — as the scores decreased, the grade level increased. However, it did not appear to be linear. The rate of grade level increase tapered, suggesting a negative exponential relationship. In line with this expectation, the best-fit exponential model had a high R^2 of 0.987 (**Figure 3**).

To summarize, we found that there is support for the hypothesis that the complexity of mathematics is higher than that of English literature in the selected middle-to-high school curriculum, with the relative change in mathematics scores between grades 5 and 7 contributing significantly.

DISCUSSION

We established that the complexity level of mathematical problems was relatively higher than that of English literature passages. A specific point to note was the sharp increase in mathematical complexity between grades 5 and 7. It would be reasonable to argue that if students cannot cope with the subject in middle school, they will continue to lag because mathematical complexity keeps increasing. At the same time, English complexity increases pace as well between grades 8 to 11. Not only would fear of mathematics have likely set in, but it would also become challenging to remediate as the students

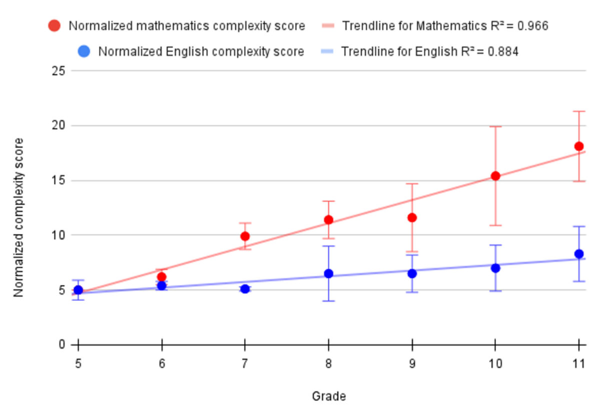

Figure 1. Normalized mathematical and English complexity scores compared. The normalized complexity scores of mathematics and English for grades 5 through 11. Circles represent the average of six samples (five samples in three instances due to outlier removal), while the error bars represent respective (±) standard deviations. The trendlines (mathematics slope = 2.13, R^2 = 0.966, English slope = 0.52, R^2 = 0.884) show a linear increase. The p-value (paired t-test, two-tailed) for the difference between mathematics and English scores was 0.012.

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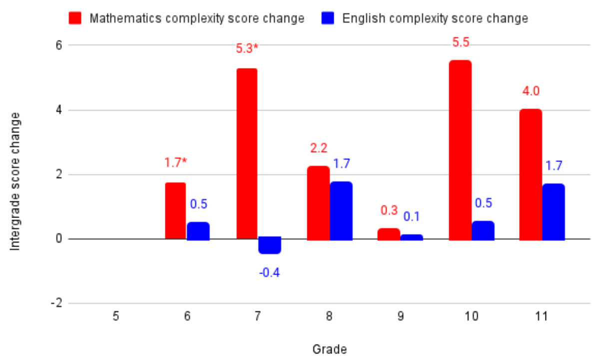

Figure 2. Change in mathematical and English complexity scores between grades. Absolute intergrade average score change for each grade (compared to the immediately previous grade) is shown. An unpaired t-test using respective data points was run to determine the significance of these changes. Statistically significant value ($p < 0.05$) shown with asterisk.

would additionally have to deal with other factors such as negative influence from parents and society or aggression from teachers for inability to solve problems in time (3, 8). Also, we introduced complexity scoring tools to help this field of research, namely, a new way of calculating mathematical complexity and a best-fit model to convert Flesch reading ease scores to complexity scores corresponding to grade levels.

While we were able to establish that a new dimension – mathematics in the context of the overall syllabus and the relative pace of complexity increase – needs attention, we do note certain limitations in our study that future work should consider for a more complete understanding of this subject. First, in terms of size limitations, there was a relatively small sample size of problems and passages, which might lead to more random variance. For mathematics, different students might solve problems differently. Thus, multiple students should independently solve the same set of problems to get a more robust idea of the number of steps. For English, Flesch reading ease scores for the entire chapters would better

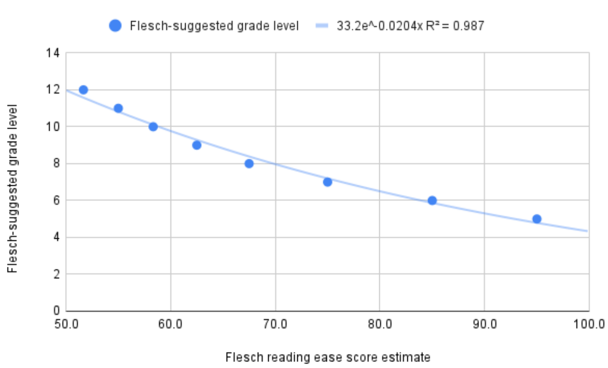

Figure 3. Flesch reading ease score versus suggested grade level. The best-fit exponential model, using interpolated values where needed, showed a high R^2 of 0.987 and provided a way to convert the reading ease scores to complexity scores (using grade level as a proxy for complexity).

indicate the difficulty level. Second, the sample space cannot be considered comprehensive. In the case of mathematics, geometry, and word problems were excluded. A method to calculate the complexity level for such problems would need to be developed. Additionally, the reading score does not fully capture the overall complexity of the English subject since poetry and grammar were not included. English is only one of several non-mathematics subjects, so a holistic understanding would also require complexity scores for those subjects. Third, we recognize a methodological limitation in cross-comparing mathematics and English complexity scores. English complexity scores correspond with grade levels as per Flesch's guidance (13). Mathematics problems were selected from grade-specific textbooks. However, we contend that the problems were not grade-appropriate. A well-structured survey of students would need to be conducted to map the problems, and thereby the complexity scores, to the grades. Finally, in terms of scope, the study focused on the curriculum from only one country; one can imagine that different countries and education boards may have different levels of mathematical complexity in comparable grades. Future studies should address these limitations.

Even as these limitations exist, our study points towards some potential solutions. Since the major increase in mathematical complexity occurs in middle school grades (5 to 7), we recommend that concepts which were introduced in grades 6 and 7 be staggered over an extended period to reduce the sudden increase in complexity (for the students who can cope up with the content, option for an advanced curriculum can be offered). This could help reduce the fear of mathematics by making it easier for students to learn and understand more straightforward concepts before moving on to more complex topics.

A typical curriculum includes five to six subjects. Mathematics gets a proportionate allocation of time. It was reported that the average time spent on mathematics in lower secondary schools in the European Union countries was about 13% (17). It could be that increasing the percentage of time spent on mathematics in school may help students grasp the concepts as they are introduced and protect against the fear of mathematics (12).

To conclude, we developed a new method to measure and score the mathematical complexity of problems. This scoring method was used with the English reading level score to compare the complexities of mathematical problems and English literature. Upon comparison, we determined that there was a statistically significant difference between the scores. This difference could be a factor in the fear of mathematics in school students. Spreading mathematical concepts over more years or increasing time spent on mathematics in school could help address this global problem.

MATERIALS AND METHODS

Data source

We used the National Council of Educational Research and Training (NCERT) textbooks adopted by the Central Board of Secondary Education (CBSE) as the primary data source for mathematical problems and English passages (18–31).

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Sampling

This study is restricted to the material for grades 5 through 11. We first accessed the online textbooks from the NCERT website to collect the literature passages and the mathematical problems. For English literature, we selected six prose chapters from each grade and extracted the first 100 words (approximately to the nearest complete sentence) from each selected chapter. For mathematics, we chose six representative questions from each grade, excluding the chapters with geometry, constructions, or proofs.

Removal of outliers

The selection of the first 100 words for English passages may not represent the complexity of the entire chapter. For mathematics, some problems can be solved in a few steps despite being higher in the order of conceptual difficulty. Thus, it was essential to test for outliers. We used the interquartile range method for identifying outliers (32). We calculated the third (Q3) and the first (Q1) quartiles using Google Sheets for the six samples of each grade for each subject. We derived the interquartile range (IQR) as the difference between Q3 and Q1 and established the fences – lower fence (LF) = Q1 - 1.5 x IQR, and upper fence (UF) = Q3 + 1.5 x IQR. We considered data below LF or above UF as outliers and removed them.

Normalization

The average complexity scores for mathematics and English were on different scales (**Table 1**). Thus, for comparison, we normalized them respectively to a complexity score of 5 in grade 5. This was done by using normalization factors on the English (5.0/6.0 = 0.83) and mathematics (5.0/7.2 = 0.69) scores for grade 5. We used this normalized data for further analysis.

Statistical analysis

A paired two-tailed *t*-test was conducted on the grade-wise average scores (from **Table 1**) to establish the statistical significance of the difference between the scores. An unpaired *t*-test using respective data points was run to determine the significance of each intergrade complexity score change. We considered a *p*-value of less than an alpha of 0.05 as statistically significant.

Calculation and graphing software

We calculated the Flesch reading ease scores using the review functionality in Microsoft® Word. We used Google Sheets for data handling, graphing, and analysis, except for the statistical test of significance, which was done using Microsoft® Excel.

Grade	English complexity score			Mathematics complexity score		
	Average	Standard deviation	Count	Average	Standard deviation	Count
5	6.0	1.1	6	7.2	0.0	6
6	6.5	0.5	5*	8.9	1.0	6
7	6.1	0.3	5*	14.2	1.8	5*
8	7.8	3.0	6	16.4	2.5	6
9	7.9	2.0	6	16.7	4.5	6
10	8.4	2.5	6	22.2	6.5	6
11	10.0	3.0	6	26.1	4.6	6

Table 1. Average English and mathematics complexity scores, standard deviations, and counts for grades 5 through 11. Six samples were taken for each of the grades for each subject. The asterisk indicates that one sample each was removed using the interquartile range method.

Scoring mathematical complexity

We propose that there are two critical determinants of mathematical complexity (y): conceptual difficulty (w) and computing difficulty (x). Conceptual difficulty (w) relates to the pre-requisite level of knowledge required to understand and solve the problem. Our reference textbooks follow the same logic in sequentially introducing the concepts as the students move up the grades (18-24). Following the same, we ordered the concepts encountered by students up to grade 11 to create an ordinal scale – earlier concepts getting a lower score and the latter getting a higher score (**Table 2**).

Given a particular level of conceptual difficulty, the computing difficulty (*x*) is dependent on the number of steps (*n*) required to arrive at a solution—the more the number of steps, the greater the computing difficulty. The computing difficulty cannot be taken as linearly proportional to the number of steps because the contribution to computing difficulty, though positive, tapers for every new step added. Thus, the rate of change of computing difficulty with respect to the number of steps is positive but inversely proportional to the number of steps, that is,

$$\frac{dx}{dn} \propto \frac{1}{n}. \quad \text{(Eq. 1)}$$

Therefore,

$$\frac{dx}{dn} = k \times \frac{1}{n}. \quad \text{(Eq. 2)}$$

where *k* is a constant of proportionality. Since the scale of complexity that we intend to develop is relative and not absolute, we can take any value for *k*. For convenience, we take *k* = 1, yielding,

$$\frac{dx}{dn} = \frac{1}{n}. \quad \text{(Eq. 3)}$$

We reorganize Eqn. 3 to

$$dx = \frac{1}{n} dn. \quad \text{(Eq. 4)}$$

and integrate it,

$$\int dx = \int \frac{1}{n} dn. \quad \text{(Eq. 5)}$$

to find the relationship

$$x = ln(n) + K, \quad \text{(Eq. 6)}$$

where *K* is a constant of integration. For *n* = 1 (just writing the problem), computing complexity *x* = 0. Also, ln(1) = 0. Thus, *K* = 0, which leads to

$$x = ln(n). \quad \text{(Eq. 7)}$$

Having established the metrics for the conceptual difficulty (*w*) and the computing difficulty (*x*), we needed to assess how they combine to give the overall mathematical complexity. Clearly, *w* and *x* do not depend on each other;

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	Ordinal scale	First introduced in	
Concept Category	Conceptual Difficulty (w)	Grade	Chapter(s)
Addition	1	1	3
Subtraction	2	1	4
Place value	3	1	5
Multiplication	4	3	9
Division	5	4	9
Decimal and fractions	6	5	4,10
Variables	7	6	11
Exponents and basic identities	8	7	12,13
Functions and advanced identities	9	10	8
Complex numbers	10	11	5
Limits and derivatives	11	11	13

Table 2. An ordinal scale of mathematical concepts and corresponding conceptual difficulty (*w*). The ordinal scale follows the order of introduction of these concepts in the standard textbooks (18-24).

a problem of greater conceptual difficulty can have a high or low number of steps to solve. Both these factors should have an increasing impact on the final score; thus, subtracting or dividing type relationships are ruled out. We cannot simply add the components because if there is only one step (that is, writing down the problem), it would have a non-zero mathematical complexity value, which it should not. We cannot use the exponential w^x as it would result in a non-zero mathematical complexity for just writing down the problem. Finally, the relationship should be practical and serve the purpose of creating the score. Thus, we take the mathematical complexity simply as the product of the two components,

$$y = w \times x \quad \text{(Eq. 8)}$$

Substituting the value of *x* from Eqn. 7, we get the formula,

$$y = w \times ln(n) \quad \text{(Eq. 9)}$$

Problem number	Solution	Conceptual difficulty (w)	Steps (n)	Complexity score ($y = w \times ln(n)$)
7.4.3.1.g	1. 7m + 19/2 = 13 2. 7m + 19/2 - 19/2 = 13 - 19/2 3. 7m = 13 - 19/2 4. 7m = 2×13/2 - 19/2 5. 7m = (26 - 19)/2 6. 2×7m = 2×(26 - 19)/2 7. 2×7m = 7 8. 2m = 1 9. m = 1/2	7	9	15.38
11.13.2.11.vii	1. d(2 tan x - 7 sec x)/dx 2. d(2 tan x)/dx - d(7 sec x)/dx 3. 2 × d(tan x)/dx - d(7 sec x)/dx 4. 2 × d(tan x)/dx - 7 × d(sec x)/dx 5. 2(sec^2 x) - 7 × d(sec x)/dx 6. 2(sec^2 x) - 7(sec x)(tan x) 7. (sec x)×[2(sec x) - 7(tan x)]	11	7	21.41

Table 3. Two sample questions with their solutions, conceptual difficulty score, number of steps, and complexity scores. Problem number is shown as: Grade. Chapter. Number (20,24).

Passage number	Text	Flesch score (W)	Complexity score ($Y = 33.2 \times e^{-0.0204W}$)
6.5	102 words, from “I had heard a great deal…” to “…yet kindly and understanding.”	80.9	6.37
11.3	115 words from “An angry wind stirred up…” to “… underground to pay their respects.”	55.4	10.72

Table 4. Two sample texts with their respective Flesch reading ease and complexity scores. Passage number shown as: Grade. Chapter and take from references 26 and 31.

that we used to calculate mathematical complexity scores. (Examples, **Table 3**)

Scoring English complexity

Though other approaches are available, such as the TOEFL (Test of English as a Foreign Language) reading assessment method, we used the Flesch reading ease test since it is a well-known method and can be easily estimated through available software (33). Flesch suggested a correspondence between reading ease scores and grade levels (13). Using these recommended grade levels as a proxy for complexity, we used the equation from the exponential model,

$$Y = 33.2 \times e^{-0.0204W} \quad \text{(Eq. 10)}$$

to convert the reading scores (*W*) to complexity scores (*Y*). (Examples, **Table 4**)

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