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Article

Estimating the elastic modulus and bending stiffness of steel ruler with crack using three-point bending test

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SUMMARY

Cracks are the early signs of distress in structures and must be diagnosed accurately. They reduce the structural stiffness, strength, and may adversely affect the behavior of the structure. This study aims to estimate the elastic modulus and the bending stiffness of a steel ruler with varying crack lengths. A low cost and simple three-point bending test was used for the measurements. A pristine ruler and rulers with three different crack lengths were subjected to loads of up to 6 kg (roughly 13 lbs) and the displacement close to the load location was measured. The load and displacement at the center of the specimen, for loading and unloading, were measured at equal load steps. The effect of a crack on the elastic modulus and the bending stiffness was compared with that of the pristine ruler. Permanent deformation was observed on the 10 mm and 15 mm crack ruler. The presence of crack reduced the elastic modulus of the ruler by 14% and the bending stiffness by 26.5%. The structural load-bearing capacity depends on the presence of cracks. Anticipating the extent of reduction in elastic modulus and bending stiffness aids designers in minimizing the need for repeated design revisions. This investigation supports both structural designers in designing structures such as airplanes and bridges, as well as repair technologists in upgrading existing structures.

INTRODUCTION

Many varieties of material are used in engineering applications. The choice of what materials to use is usually based on their stiffness and strength, which are the two fundamental properties of materials. Stiffness is the ability of the material to resist deformation and is inversely proportional to deformation (1). However, the response to deformation is characterized by the elastic and plastic behavior of materials. Elastic behavior refers to reversible deformation in response to small stresses, while plastic behavior involves irreversible deformation under higher stresses that cause permanent changes in the material's shape. The transition between these two behaviors is typically marked by the yield strength of the material. Manufacturers furnish yield strength and elongation values corresponding to diverse heat treatment processes for various material alloys (2). The stiffness of a material is often measured as its elastic modulus. The material's elastic modulus is generally indicated for a specific heat treatment alone. Nevertheless, there exists considerable variability in elastic modulus due to different heat treatment processes and applied coatings. Unfortunately, manufacturers often overlook these variations (2-4). Consequently, designers are compelled to use a uniform elastic modulus value regardless of the material's specific heat treatments and coatings. The elastic modulus of a material directly impacts its bending stiffness, which is the resistance of material to bending deformation. Bending stiffness of a beam also depends on the area moment of inertia, a property of a two-dimensional plane shape which characterizes its displacement under loading. Consequently, both elastic modulus and bending stiffness assume a significant role in the design of structures like bridges and aircraft wings, where constraining deformation to specific limits is essential.

Steel is widely used as a building material. The preferred failure mode in steel structures is ductile rather than brittle. This choice is primarily due to the fact that brittle failures carry a substantially greater level of danger, posing a significant threat to human life within such structures. A brittle failure occurs instantaneously, without any warning, whereas a ductile failure in a structure gives ample time before collapse (5). Furthermore, the structure should be designed to reduce failure during operation. Cracks in any structure will lead to catastrophic failure over a period of time. Steel can crack during welding, in fatigue, and due to corrosion. Apart from this, cracks can be initiated during transportation and handling. The strength of material is drastically reduced due to the presence of cracks (6,7). Many structural failures have occurred due to the presence of cracks in structures. The growth of cracks during the lifetime of structures results in their collapse. Structural failures like the Boston molasses tank failure, the Liberty ship failure, and the Comet aircraft failure have taught lessons on fracture mechanics (8). For metallic materials, the effect of crack growth in fatigue has been widely studied (9,10).

Previous work has performed analysis of free vibration, a type of vibration where a force is applied, and the structure is allowed to vibrate at its natural frequency. The researchers conducted this analysis on beams with cracks of varying geometry, depth, and location (11). They also measured the stiffness of a beam using displacement and vibration methods, for varied location and depth of cracks. They concluded that the stiffness of the beam was sensitive to change in crack geometries. Another paper studied the effect of density and radii of the cracks on the elastic moduli of brittle material (12). They obtained the empirical relations between elastic modulus and shear modulus for cracked and un-cracked brittle material. Limited research has addressed the changes in elastic modulus due to the presence of cracks in ductile materials.

The traditional method to measure the elastic modulus of a material is to perform a tensile test in a universal testing machine or by using ultrasonic waves (13-16). These methods result in an accurate measurement of the elastic modulus, but they heavily depend on expensive equipment and a complicated test setup. In this study, we propose a low-cost and simple method to measure the elastic modulus of the material. We hypothesized that a crack in stainless steel ruler would reduce the elastic modulus and the bending stiffness. In the present study, we used a low-cost and simple three-point bending test to estimate the elastic modulus of pristine and cracked rulers. Cracks of three different lengths – 5 mm, 10 mm, and 15 mm - were tested. Our study aimed to provide accurate estimations of the reduction in the bending stiffness of the cracked ruler.

RESULTS

To determine the elastic modulus of a stainless-steel ruler, we used three-point bending method. This method is universally used to measure the fracture toughness of specimens (17). Generally, the specimen in the three-point bending method is supported in the universal testing machine with a three-point bending fixture. However, in our study, a fixed boundary condition was used to avoid slipping of the stainless-steel ruler during application of the load (**Figure 1**). The specimen considered for this study was a 320 mm long, commercially available stainless-steel ruler with a width of 25 mm and 1 mm thickness. To simulate cracks, cuts of varying lengths - 5 mm, 10 mm, and 15 mm and a width of 1 mm - were made at the center of the ruler using a handheld hacksaw frame (**Figure 2**). As the cut represents a crack in this study, it will be referred to as such from here onward. We applied load at the center of the ruler and measured displacement close to the center.

Figure 1: Experimental setup of three-point bending test. A) Test setup showing the boundary conditions. **B)** Displacement measurement. Steel ruler was clamped at the ends to the table using C clamps. Mass was hung at the center of the ruler using an S hook. A 600 mm long scale was used to measure the displacement. A pin was attached to the center of the ruler, which was used as a pointer for measuring the displacements**.**

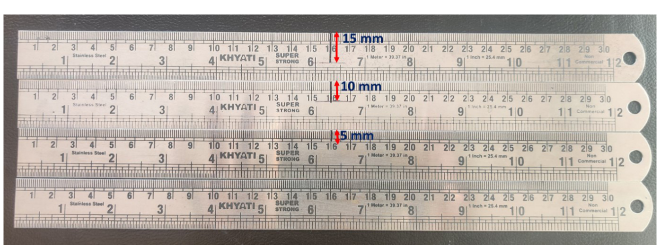

Figure 2: Pristine and varying crack length rulers. A 300 mm long, 25 mm wide, and 1 mm thick stainless-steel ruler was used for the experiments. 5 mm, 10 mm, and 15 mm long rectangular cracks were made at the center of the rulers. Cuts with a width of 1 mm was made in the ruler using a handheld hacksaw frame to simulate cracks.

The load versus displacement relationship exhibits a linear behavior for the stainless-steel ruler in its pristine condition, without any cracks (**Figure 3**). We made the measurements on three different pristine samples and plotted the mean displacement value. We carried out the experiment on three samples for each crack width keeping all the conditions identical. We used the mean displacement value for plotting the results.

We conducted two sample two tailed t-tests to check if there was significant change in the displacement for each cracked ruler, compared to the pristine ruler. The pristine ruler had a displacement of 18.6 mm, whereas the ruler with a 5 mm crack exhibited a displacement of 19.5 mm ($p=0.7$). The 10 mm crack ruler had a displacement of 22.25 mm ($p=0.1$) whereas 15 mm cracked ruler had a displacement of 24.5 mm ($p=0.03$). However, no significant change was observed when 10 mm cracked ruler was compared with 15 mm cracked ruler ($p = 0.4$).

We calculated the stiffness of the ruler from the slope of the load versus displacement curve. The pristine ruler had a stiffness of 0.33 kg/mm. Linear behavior was observed in ruler with 5 mm crack until 4.5 kg load similar to the pristine ruler (**Figure 3**). However, after 4.5 kg of load, the stiffness reduced to 0.25 kg/mm.

In the case of the 10 mm crack ruler, there was a reduction in stiffness from the initial loading when compared to the pristine ruler. The stiffness of the 10 mm crack ruler measured 0.285 kg/mm until reaching a load of 4 kg, after which it decreased to 0.25 kg/mm. The stiffness of the 15 mm crack ruler was 0.285 kg/mm, similar to the 10 mm crack ruler until 2.5 kg load. After 2.5 kg load, it was further reduced to 0.222 kg/mm. The elastic modulus of the pristine ruler and the ruler with a 5 mm crack was calculated to be 199.4 GPa. The elastic modulus of the 10 mm and 15 mm cracked rulers were calculated to be 170.9 GPa.

We also measured the displacements while unloading the rulers (**Figure 4**). All the specimens were unloaded in the

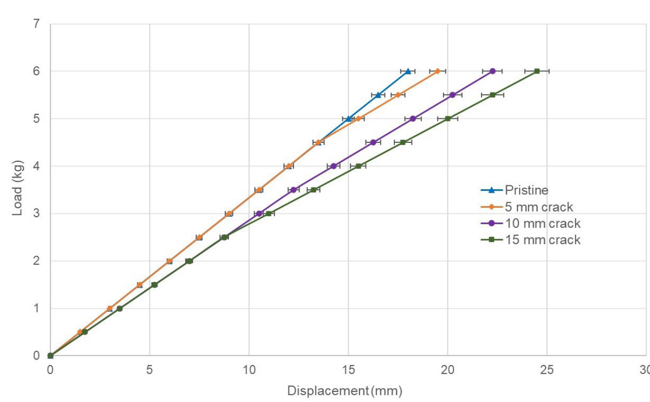

Figure 3: Load-displacement curve of all rulers. Load was applied to the ruler in steps of 0.5 kg to a maximum of 6 kg. The displacements at each load step were measured using a 600 mm ruler. The slope of the curve is a measure of the stiffness of the ruler. N = 3, Mean ± SE. A significant change was observed for the pristine and 15 mm cracked ruler ($p=0.03$). The t-tests were conducted using GraphPad software.

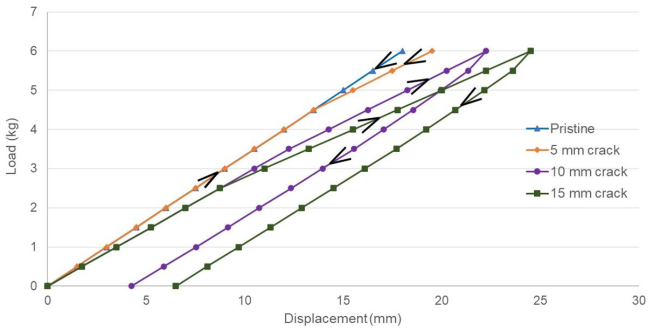

Figure 4: Loading and unloading of rulers. The loading and unloading responses followed different paths in the rulers with a 10 mm and 15 mm crack, indicating permanent deformation. No permanent deformation was observed in pristine and 5 mm crack ruler. The arrows show the loading and unloading path.

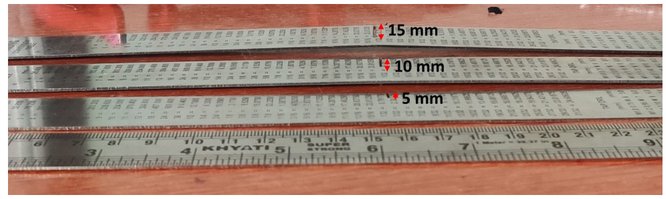

Figure 5: Permanent deformation in rulers. The rulers with a 10 mm and 15 mm crack had permanent deformation of 4.25 mm and 6.5 mm, respectively. No permanent deformation was observed in pristine and 5 mm crack ruler.

same equal steps as loading. Both the pristine and 5mm crack ruler behaved elastically and followed the same unloading path as their loading path. However, the rulers with the 10 mm and 15 mm cracks had 4.25 mm and 6.5 mm of permanent deformation (**Figure 5**).

To understand the effect of permanent deformation on the elastic modulus the 15 mm crack ruler, which had a 6.5 mm permanent deformation, was loaded again (**Figure 6**). Permanent deformation further decreased the stiffness of the ruler. The ruler was loaded only until 4 kg as the displacements were higher than expected.

DISCUSSION

The aim of this study was to determine the effect of varying crack lengths on the elastic modulus and the bending stiffness of a stainless-steel ruler. To simulate cracks, rectangular cuts of width 1 mm and lengths 5 mm, 10 mm, or 15 mm were made on the steel ruler. The present study was limited to the rectangular cuts of three different lengths to represent cracks. The displacement close to the center of the ruler was measured using the three-point bending method up to a load of 6 kg.

The load versus displacement curve for the pristine ruler exhibited linearity, indicating that the pristine ruler adheres to Hooke's law and undergoes linear elongation in response to the applied load (**Figure 3**). However, the presence of crack in the ruler reduced the stiffness. Moreover, rulers with 10 mm and 15 mm cracks experienced an additional decline in stiffness.

The cut at the center resulted in a non-uniform cross section. However, for the purpose of estimating the elastic modulus, the ruler was assumed to have a perfectly rectangular section. Hence, the elastic modulus was an estimated value due to this structural change. The elastic modulus of commercially available stainless steel was reported as 200 GPa (18). The measured value of the elastic modulus of the pristine ruler and the ruler with 5 mm crack was close to the reported value in the datasheet. There was a 14% reduction in the elastic modulus of the 10 mm and 15 mm cracked rulers when compared with the pristine ruler. The cracked rulers exhibited greater displacement at the same load in comparison to the pristine ruler (**Figure 3**). Furthermore, 10 mm and 15 mm cracked rulers had almost 50% material loss in the central cross section. Therefore, higher crack length reduced the elastic modulus of the rulers. As the material loss was less in 5 mm cracked ruler, there was no reduction in elastic modulus. Our hypothesis that cracks would reduce the elastic modulus of the ruler held true for crack lengths greater than 10 mm, though it proved untrue for smaller cracks.

The presence of cracks not only reduced the elastic modulus of the ruler but also reduced the area moment of inertia. With increase in crack length, the bending stiffness reduced in the elastic and plastic region (**Table 1**). Though the elastic modulus did not show considerable change, when comparing 5 mm cracked ruler to pristine ruler, bending stiffness had reduced. The bending stiffness of 5 mm cracked ruler in the plastic region reduced by 7.7% and that of 10 mm cracked ruler by 19.1%. The reduction in bending stiffness of 15 mm cracked ruler was 26.5%. We observed that the reduction in bending stiffness in the plastic region was more than the reduction in the elastic region. It was also observed that the elastic limit threshold reduced with increase in crack length.

The pristine ruler exhibited elastic behavior up to a load of 6 kg, whereas rulers with crack had both elastic and plastic response. The ruler with a 5 mm crack experienced a shift in its slope after surpassing 4.5 kg, indicating the crossing of the proportionality limit. The proportionality limit, is a point on the stress-strain curve of a material where its deformation remains linearly proportional to the applied stress. Nevertheless, it remained within its elastic limit- a point on the stress strain curve below which the material restores its original shape upon the removal of load- up to a load of 6 kg. This suggests that a greater force would be necessary to initiate plastic deformation. On the contrary, both 10 mm and 15 mm crack rulers exhibited plastic behavior.

The loading and unloading curves for 10 mm and 15 mm crack specimens had a shift due to permanent deformation. However, the unloading curve was parallel to the loading curve (**Figure 4**). This is attributed to the dislocation of the molecules in the material during plastic behavior. The elastic modulus is proportional to the interatomic distance, the distance between the nuclei of atoms in a material. The material is undergoing a rearrangement of its molecular structure. The atoms are being moved to new equilibrium position however, their interatomic distance remains the same. Hence the slope of unloading curve is parallel to the slope of the loading curve. The cracks in the ruler lead to a further reduction in the material's stiffness when it is loaded after a permanent deformation (**Figure 6**).

Generally, cracks are simulated using varied geometric shapes like U-shaped, V-shaped, and rectangular cuts, but this study was limited to the effect of cracks simulated as rectangular cuts alone. Another limitation in this study was that

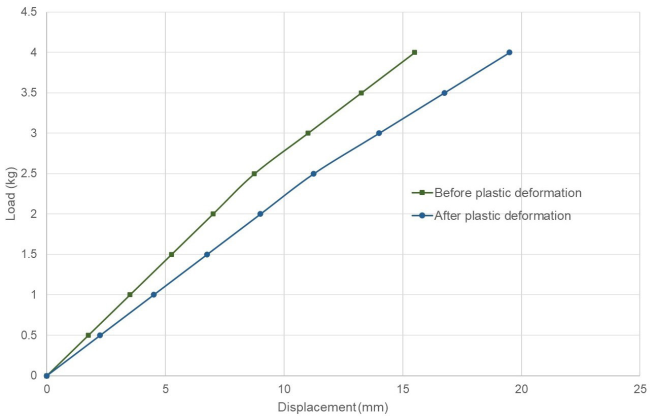

Figure 6: Load displacement curve of 15 mm crack ruler. The ruler with 15 mm crack was loaded and displacement measured before and after it underwent plastic deformation. After plastic deformation, the specimen was loaded only up to 4 kg, as the displacements were more than the expected displacements.

the displacement was not measured exactly at the center of the specimen. There was a gap of 7 mm between the loading hook and the pointer to avoid interference while measuring and loading the mass. This should not have influenced the results much as we made sure to record the displacements exactly at 7 mm from the center for all the experiments. The elastic modulus calculated using equation 1 is for beams with uniform cross-section. The effect of a cut on the specimen while estimating the elastic modulus was ignored as the specimen had a uniform cross-section throughout the length except the 1 mm wide cut. The lowest count on the ruler was 1 mm. More accurate results would have been measured if other instruments like a dial gauge or non-contact displacement sensor was used to measure the displacements. The test setup could be further improved by using a rigid work bench to avoid slippage due to higher loads.

This study can be extended to evaluate the elastic modulus and the bending stiffness when the location of the crack is varied in the specimen. The crack geometry (U-shaped and V-shaped) can be varied and its effect can be studied. Further investigation to develop a model for predicting the effective elastic modulus based on the crack size could be taken up with a focus on gathering experimental data from a range of materials and crack sizes. This data could then be used to establish relationship between crack size and its influence on the material's overall mechanical properties.

The elastic modulus and the bending stiffness of beams is of utmost importance to designers. The safety and reliability of structures depends on these fundamental parameters and the presence of a crack influences them. The life and load carrying capability of the structure depends on the presence of cracks. A structure can develop cracks at any point of time during its service. Presence of cracks in the structure can lead to catastrophic failure, if it is not noticed and repaired. Estimation of load carrying capacity of the structure in the presence of cracks can help designers in estimating the remaining life of the structure. Our cost effective and simple study is one of the methods to estimate the effect of a crack on a material's properties. It can be used in the preliminary design stage to get a quick estimate of the mechanical properties of the material.

	Pristine ruler	5 mm crack ruler	10 mm crack ruler	15 mm crack ruler
Elastic Modulus (GPa)	199.4 ± 3.9	199.4 ± 4.9	170.9 ± 4.2	170.9 ± 5.1
Bending Stiffness in elastic region (Nmm^2)	$(4.15 \pm 0.08) \times 10^5$	$(4.15 \pm 0.1) \times 10^5$	$(3.56 \pm 0.08) \times 10^5$	$(3.56 \pm 0.1) \times 10^5$
Bending Stiffness in plastic region (Nmm^2)	-	$(3.83 \pm 0.09) \times 10^5$	$(3.36 \pm 0.08) \times 10^5$	$(3.05 \pm 0.09) \times 10^5$

Table 1: Elastic modulus and bending stiffness for the pristine and cracked specimens. Elastic modulus and bending stiffness in the elastic region were calculated using the slope of the first linear region in the load displacement curve. The bending stiffness in the plastic region is calculated using the slope of the second linear region.

MATERIALS AND METHODS

Commercially available stainless-steel rulers of 320 mm length were used in this study. The steel ruler was clamped on either side using C clamps to the edges of two tables (**Figure 1**). The effective length of the specimen after clamping on the sides was 290 mm. A 600 mm long ruler was attached to a white board and placed behind the test specimen to measure the displacements. An "S-hook" was placed at the center of the ruler to hang the mass. A pin was attached close to the center of the ruler, which acted as a pointer to aid in measurement of the displacements. Twelve weights of 500 g each were added to the hook and the displacement at each step was noted. As soon as the weight was added, the ruler would deflect and there was a minor vibration due to the moving weights. Hence, the displacement was recorded only after the needle and the ruler movement stabilized. This was done to avoid any errors in recording the displacements. The ruler was loaded to a maximum of 6 kg. The measurements during unloading were also noted with the same number of load steps.

Cracks of 5 mm, 10 mm, and 15 mm were made in new rulers of the same material and dimensions. These rulers with cracks were tested under the same conditions. The displacements were measured at regular intervals for a load step of 500 g each. The maximum load applied on the rulers was 6 kg. The displacements were also measured during unloading of the specimen.

The weight acting on the ruler was calculated as $w=mg$, where m is the mass of the load and g is the acceleration due to gravity. The elastic modulus for the three-point bending

$$E = \frac{wl^3}{192\delta I} \quad \text{(Equation 1)}$$

specimen is:
where w is the applied force, l is the length of the ruler, δ is the displacement at the center and I is the area moment of inertia (19). For a rectangular, cross-sectional area, the moment of

$$I = \frac{bd^3}{12} \quad \text{(Equation 2)}$$

inertia is:
where b is the breadth and d is the depth of the cross section (19). The bending stiffness of a beam is the product of the elastic modulus and the area moment of inertia. Hence, the

$$EI = \frac{wl^3}{192\delta} \quad \text{(Equation 3)}$$

bending stiffness of a three-point bending specimen is:

The slope of the load versus displacement curve gives the stiffness of the ruler. The slope of the first linear curve was used for calculating the elastic modulus using equation 1. The slope of the first linear curve and the second linear curve was used for calculating the bending stiffness using equation 3.

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