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Article

The most efficient position of magnets

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SUMMARY

People mount materials ranging from paper to wedding photo frames, including school notices to the refrigerators every day at home. The position of the mounts used is essential for both safety and durability. We investigated the most efficient way of positioning magnets that would hold the most pieces of paper on the surface of a refrigerator. We estimated our model coefficients using a regression model with the help of an artificial neural network and decision tree and focused specially on the feature importance measure. We found that when the four magnets are placed in symmetry while the rectangle formed (by drawing a line between the four magnets so that the four magnets become the vertices of the rectangle) contained the center of gravity of the papers, the magnets held more paper. The most efficient points of four magnets predicted in our analysis are in between the center of gravity of the papers and four centers of gravities of four quadrants of a paper. Our study not only considers conditions of efficiency, but also safety, aesthetics, and facility of comprehension. Our findings provide insight into magnet efficiency as resources are scarce, safe for applications, aesthetically pleasing, and easy for individuals to comprehend.

INTRODUCTION

Mounting and attaching items from paper to flat-screen monitors to walls and other flat surfaces is a common occurrence in daily life. Traditionally, people drill holes in walls and use bolts in order to secure heavy materials. However, it takes a lot of labor to drill holes in solid walls, and it makes it extremely difficult to change the locations of materials once mounted. Therefore, we investigated the use of magnets as an alternative means to mount materials on vertical magnetic surfaces. Our method consisted of using magnets to hold papers on a refrigerator to investigate the most efficient way of positioning magnets.

There are two causes for the paper to fall from the refrigerator: gravity load and torque. The gravity load causes vertical motion, whereas the torque causes rotational motion **(Appendix Figure 1)**. Torque occurs as the magnets move away from the center of gravity due to eccentric force which produces rotation. In eccentric shear, the shear force does not pass through the center of gravity of the four magnets (1). This results in a torsional moment (torque) on the magnet group (1). Torque is calculated using the following equation:

$$Torque = r \times F \tag{1}$$

Where r is the position vector (i.e., distance to its axis of rotation) while F stands for the force vector. The equation of forces below **(Eqn 2)** shows the relationship between the static friction force and the normal force. The normal force in this equation is the reaction force acted upon the magnets by the refrigerator due to the force of the magnets pushing on its surface. Therefore, the equation below **(Eqn 2)** is related to gravity load. As shown in the equation below, for the maximum number of papers to be held in place without falling, the maximum static friction must be maximized, thus the normal force must be maximized in order to counter the downward motion due to gravity load.

$$Static\ friction \leq Normal\ force \times Coefficient\ of\ friction \tag{2}$$

Even after having the center of gravity for four magnets aligned with that of the papers, when the four magnets are not symmetrical, they held less paper due to the resulting torque (1). The magnitude of the load that each magnet receives is proportional to the linear distance from the centroid. The fall of a body occurs at the location of the weakest mount among multiple mounts supporting the body (stress concentration) (1). Therefore, the distance between the magnet and the center of gravity of the papers must be equal for all four magnets to level out or equalize in load (1). Also, if the moment and the location of the four magnets are given and fixed, the moment is inversely proportional to the sum of the squares of the linear distances from the centroid of papers to the center of each magnet (2). Thus, the four magnets must be placed at a considerable distance from the center of gravity of the papers (2). One study performed a regression analysis using compressive stress as an independent variable and the distance between the bolt and the friction surface where the deformation occurred due to the bolt (3). Our study also utilized regression analysis. A linear regression model can interpret the results straightforwardly and can accommodate non-linear forms such as logarithmic, quadratic, and cubic.

The relationship between the positioning of magnets and the maximum number of papers held without falling may be more accurately represented in logarithmic form within a regression model (3, 4). It has been shown that the relationship between stress range and the number of cycles to failure is linear when each variable is expressed in logarithmic form (3, 4). Therefore, the relationships between variables in this study can be better represented in non-linear forms. In addition, the distance in our study is not singular. The distance can be the distance between the center of gravity of the papers and the four magnets, the distance between the four magnets,

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distance from the corners, or the distance from the center of gravity of the four quadrants formed when the papers are folded two times. Furthermore, the sum of the linear distances, the sum of the squares of the linear distances, or the square of the sum of the linear distances may have a more explanatory power. Differing from a previous study, we decided not to consider cube of the distance (5). In principle, the square of the sum of the linear distances has the most information, including cross product (covariance in statistics) between terms. Therefore, our model consists of a linear or logarithmic model in which the diverse distance variables and independent variables were input.

Our hypothesis was that the four magnets must contain the center of gravity of the papers to hold more paper. The center of gravity is the imaginary point of an object where the body's total weight is concentrated (6). This point helps in designing static structures and in predicting the movement of a body that is acted on by the force of gravity. When mounting an object on a vertical surface, the center of gravity of the object must be supported by the mounts fastening it to the vertical surface to minimize a torque generation **(Eqn 1)** (1). Therefore, the center of gravity of the mounts must be as close as possible to that of the object being mounted. Otherwise, the object would get damaged by the serious eccentric force (1).

We tested our hypothesis by using regression analysis with the help of Artificial Neural Network (e.g., Multilevel perceptron, MLP) and the decision tree (e.g., Classification and regression tree, CART) methods (7, 8). Artificial neural networks inclusive of MLP (Multilayer perceptron) are the preferred tool for many predictive data mining applications. The MLP is a sort of supervised machine learning method (7). The Decision Tree procedure inclusive of CART (Classification and regression tree) predicts values of a dependent variable based on values of independent variables. The procedure provides validation tools for exploratory and confirmatory classification analysis (8). Economically speaking, four identical magnets (cost of production) produced results that differ from two to fourteen papers (output). The points that hold 14 papers are the most efficient points in an economical sense. In summary, the locations of the four magnets with maximum efficiency are near four CG(i), that is, the centers of gravity of four quadrants of a paper. Future research on the dynamic position change of the four legs of a robot should expand our research, which is static.

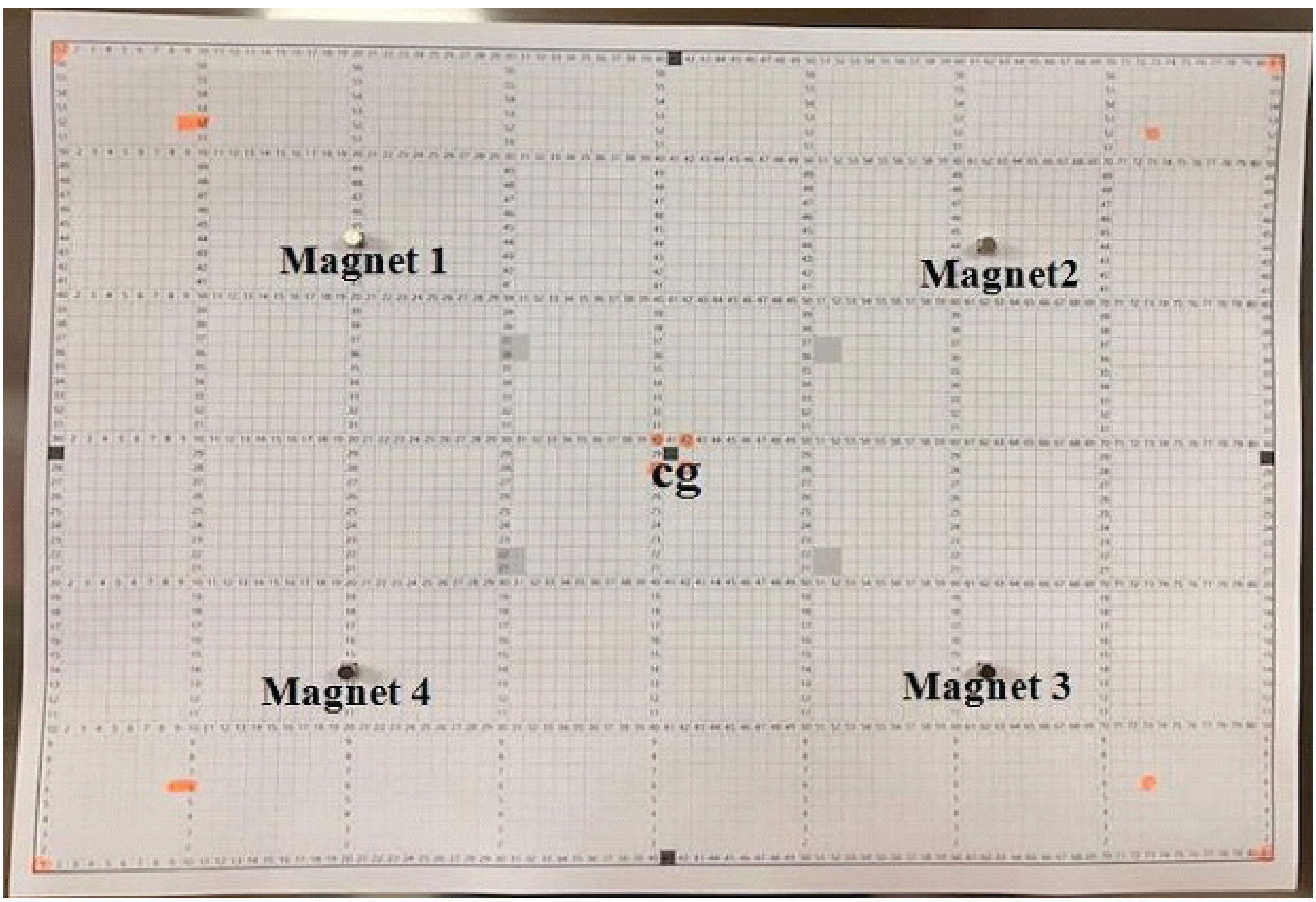

Figure 1: Experiment Set-up for Magnet Arrangement 4. We created our own Coordinate system on an A4 paper; (x, 1 to 81) (y, 1 to 57). First, we determined the location of five important centers of gravity; One cg and four CGs. The cg (41, 29) is the center of gravity of the paper while the four CGs, where the magnets are placed on the figure, from CG(1) to CG(4), are the centers of gravity of four quadrants of the paper, where quadrant 1-left upper quadrant, quadrant 2, quadrant 3 to quadrant 4, clockwise, respectively. Our coordinate value is not a point, but a rectangle with area. Therefore, the coordinate value is not accurate but an approximation. The highlighted and shaded areas are some of the magnet arrangement positions tested throughout our investigation.

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RESULTS

To test the best position of magnets to hold more papers in a safer fashion we arranged 4 magnets in 50 different ways and tested how many papers each magnet arrangement could hold up **(Figure 1)**. The maximum value of our dependent variable (Number of sheets of papers) is 14, with a minimum of 2 and an average of 9.6. Out of the 50 tests, 66% of the magnet arrangements failed between 12 and 13 papers **(Figure 2)**. In the case of 2 papers, the magnets were concentrated at a corner, which probably resulted in large eccentricity. The maximum number of papers, 14 papers, was achieved near the 4 centers of gravity (CG(i)) **(Appendix Figures 2 and 3)**.

To find a functional relationship between the number of papers and four locations of magnets, we regressed the dependent variable on the various distance measures of magnets. With this fitted linear function, we can predict the number of papers on the possible positions of four magnets. We input important independent variables (or a group of variables) one by one to fit multiple linear regression models **(Table 1 and Appendix Table 1)**. The overall regression was statistically significant (R^2 = 0.966, $F(6, 43)$ = 100.76, p = 0.01). This R^2 value ranges from 0 to 1. The significant input variables are tot_sum_CGSQ, tot_cg, dist2CG, dist3CG, and tot_sum_cgsq. The RMSE of BLM is 0.9165 and the prediction error is less than one paper.

We tested the first hypothesis of center of gravity effect via a statistical *t*-test method. We performed an average *t*-test on the group containing the center of gravity of the paper and the group that does not contain the cg **(Figure 3)**. We used 45 cases excluding outliers (4 cases where only 2 papers were held, and one case where all 4 magnets were stacked on top of the other like a tower on the cg) to perform the test. Magnet arrangements that surround the CG were able to hold significantly more papers than arrangements that did not surround the CG (n = 45, *t*-test, p = 0.01) **(Figure 3)**. Therefore, containing the cg allows the magnets to hold 1.85 more papers. Although our BLM is the best among linear models, that doesn't guarantee BLM is better than non-linear models. To confirm the robustness of BLM, we decided to do a model comparison test with non-linear models such as MLP and CART. The BLM is again robust. We have also reported the results of the comparison between input variables of the competing models **(Figure 4)**.

After finding the maximum point from our estimated regression model, we determined how the position of Magnet 2 (the upper right magnet) affects the predicted number of papers the arrangement can hold **(Figure 5)**. We showed the

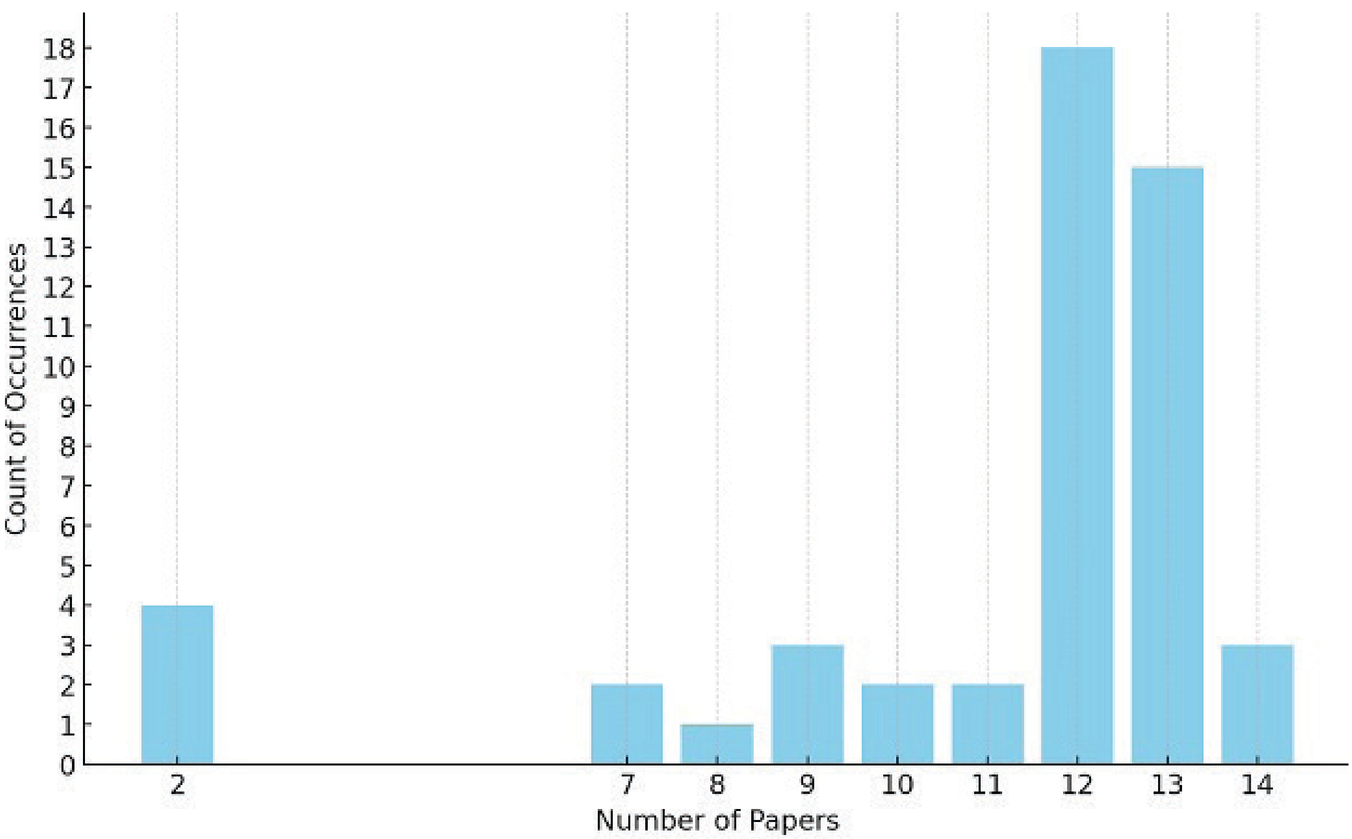

Figure 2: Distribution of number of papers held by four magnets. We prepared blank A4 papers (4.7–5g) and selected 50 different positions in which we placed 4 circle-type magnets with a radius of 2mm in the shape of a rectangle. The possible set of positions of four magnets were 4617 P 4 = 4617*4616*4615*4614. It was not possible to try all of the positions. We decided to experiment with only rectangular polygons using four magnets in up to 50 cases. The number of blank A4 papers between the surface of the refrigerator and the magnets in each position was increased by 1, and the maximum number of blank A4 papers each position could hold without slipping was recorded. The maximum number of papers held was 14 (3 cases), minimum number was 2 (4 cases), and the average was 9.6 papers. Out of the 50 tests, 66% of the magnet arrangements failed between 12 and 13 papers.

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R	Adjusted R^2	RMSE	
0.966	0.926	0.925	
Variables	Coefficient	Standard error	t
(constant)	15.415	0.585	26.334
$dist_2CG$	-0.069	0.016	-4.421
$dist_3CG$	0.071	0.015	4.625
tot_cg	-0.054	0.010	-5.177
tot_sum_cgsq	0.001	0.000	6.052
tot_sum_CGSQ	-0.001	0.000	-16.378

Table 1: The best linear model (BLM) with respect to the adjusted R^2. Input variables are constant, tot_sum_CGSQ, tot_cg, dist2CG, dist3CG, tot_sum_cgsq and dependent variable is the Number of papers. RMSE = root mean squared error.

coordinates of Magnet 2 because all other magnets are in symmetry and containing the CG in this analysis. We found the maximum number of papers held for a given position of Magnet 2, as predicted by the BLM, through the Trial-and-error method by using Microsoft Excel program. The maximum number of papers is 12.608, where $(x_2, y_2) = (54, 36)$ **(Figure 5, Appendix Figure 3)**. This coordinate of maximum point is located between CG(2) (62, 44) and CG (41, 29) **(Figure 5)**.

DISCUSSION

We found that when we placed four magnets placed in symmetry while supporting the center of gravity for the papers, they could hold more papers. The optimal points of four magnets predicted in our analysis were between the center of gravity of the papers and four centers of gravities of four quadrants of a sheet of paper. In our study, we used our hands and eyes to do our analysis; however, future studies can be done with special analysis programs. The distance variable is essential to achieve the aim of this study, the most efficient position of four magnets. Furthermore, the nonlinear effects that may exist between distance and the number of papers can be sufficiently analyzed with methods such as

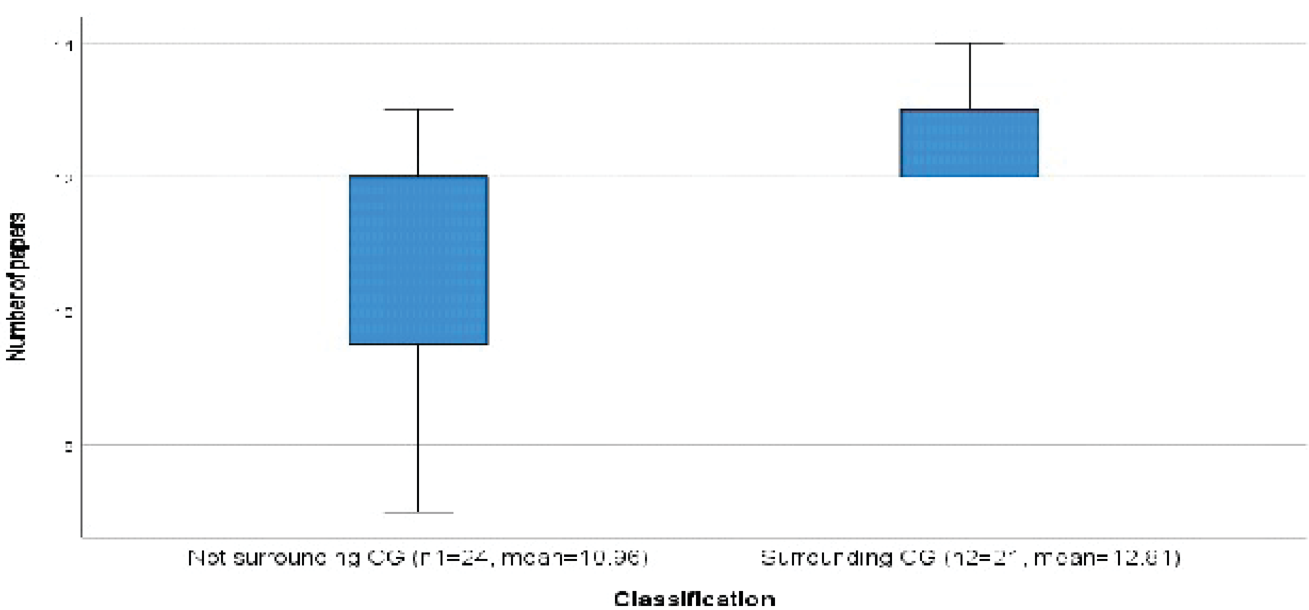

Figure 3: Difference in mean between two sub-groups *t*-test is statistically significant. Magnet arrangements that surround the CG were able to hold significantly more papers (1.85 papers) than arrangements that did not surround the CG (n1+n2 = 45, t=4.28, p = 0.01).

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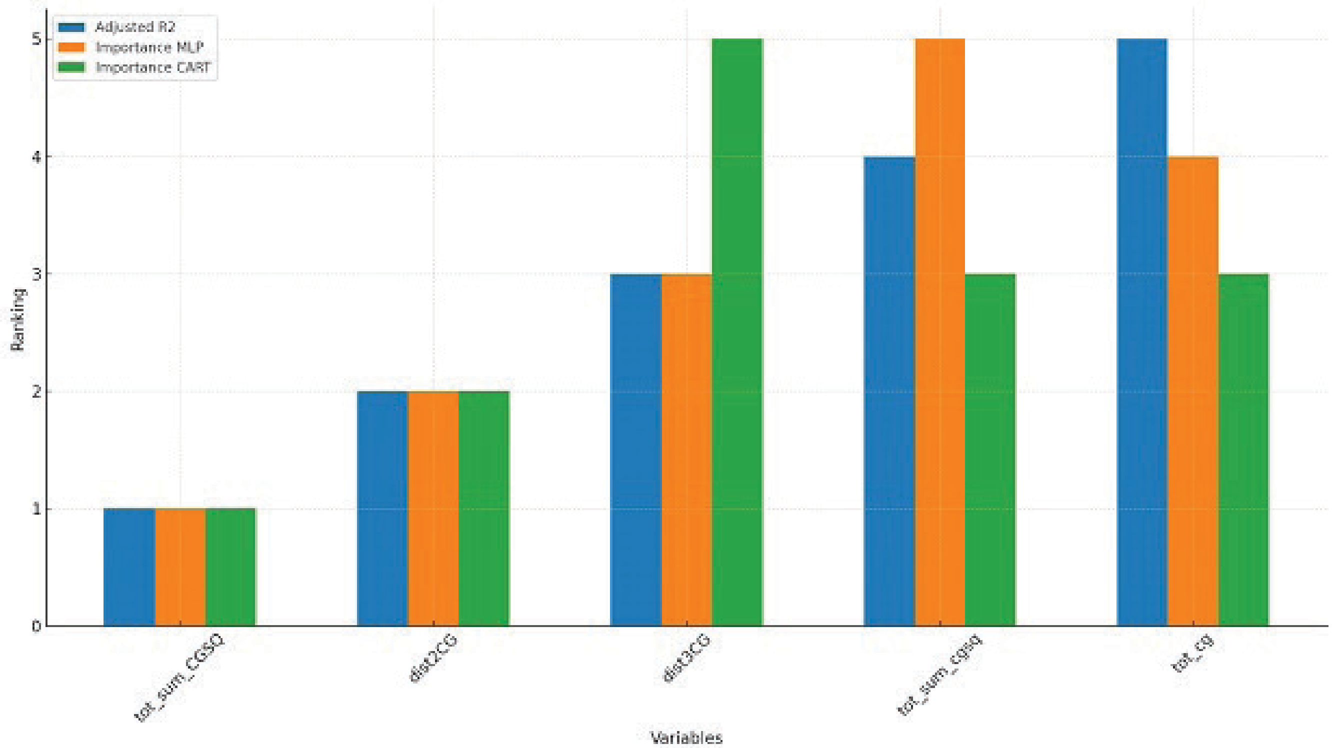

Figure 4: Importance rankings of variables (features) among competing models. The y-axis of the figure shows the importance rankings of independent variables. The importance ranking of independent variables of the BLM (linear model) is quite similar to those inputted in the non-linear machine learning models such as MLP and CART. The tot_sum_CGSQ and dist$_2$CG (the left two) are the top two important variables (features) for all models. For all three models, tot_sum_CGSQ and dist2CG ranked first and second in terms of importance in determining the efficiency of magnet arrangements, and dist3CG, tot_sum_cgsq, and tot_cg ranked 3rd, 4th, or 5th for each model.

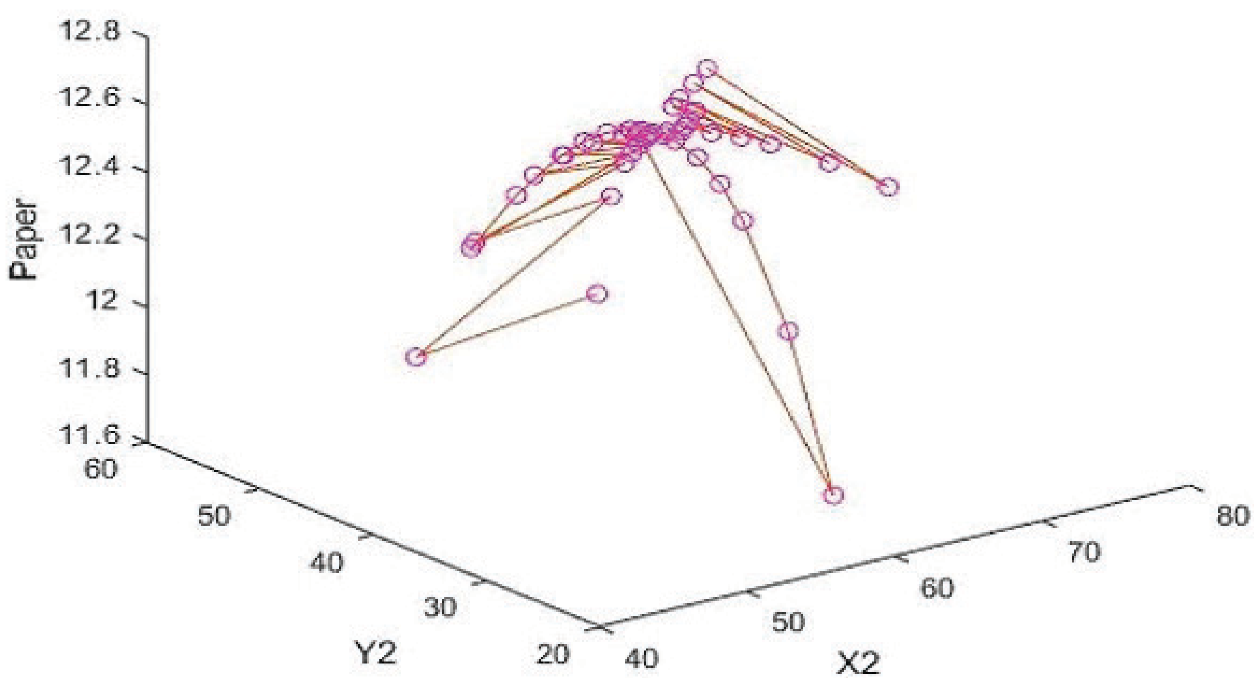

Figure 5: The BLM predicts that the maximum number of papers is held when Magnet 2 is at (54, 36). Predicted number of papers held in place for indicated positions of Magnet 2, the upper right magnet in the rectangular arrangement. x_2 and y_2 are the X (1 to 81) and Y (1 to 57) coordinates of Magnet 2, respectively. The Microsoft excel trial and error method was used to predict the number of papers held for each position of Magnet 2.

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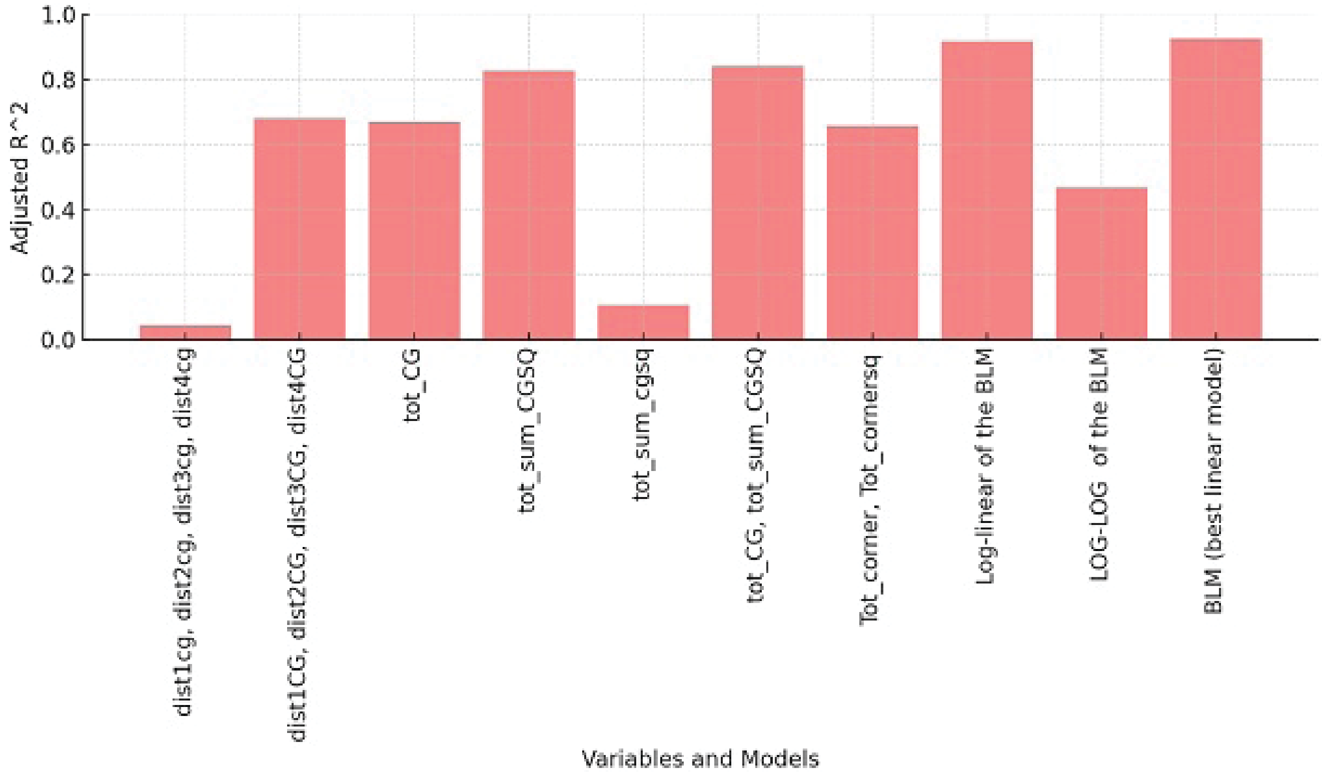

Figure 6: The competing model comparison. Model is directly implying the input independent variable(s), X-axis. Y-axis measures adjusted R^2. The order of models is distance variable (e.g., disticg) first, then derivatives such as tot_cg, then log transformation, then BLM (R2=0.926). Log-linear model (R^2=0.917) means just log-transforming the dependent variable only, while LOG-LOG (R^2=0.467) means transforming both the dependent and independent variables of our BLM.

logarithmic function transformation and squaring the diverse distance variables.

Our hypothesis can be explained by the BLM determined by regression analysis **(Table 1)**. The coefficient of tot_sum_CGSQ (sum of squared four distances between 4 CG(i) and 4 matching magnets) is negative and statistically significant. This variable measures the variance in statistics and the unequal distance from the CG(i) in our analysis. Therefore, the maximum number of papers held decreases as the variance of the 4 magnets' distance to CG(i) increases. The coefficient of tot_cg and the coefficient of tot_sum_cgsq should be inferred together simply because they share the common property of the distance from the cg. The coefficient of tot_cg (sum of four distances between magnets and cg) is negative while the coefficient of tot_sum_cgsq (sum of squared four distances between cg and magnets) is positive. Putting this all together, when moving away from the cg periphery, the coefficient of tot_cg (B = -0.054; p = 0.01) has a large influence. Therefore, the number of papers decreases. However, when moving toward the corner, the influence of tot_sum_cgsq (B = 0.001; p = 0.01) is greater, and the number of papers increases.

Putting the above three variables together, the optimal points were interpreted to exist between CG(i) and cg. The cg is the center of gravity of papers while the four CGs, from CG(1) to CG(4), are the centers of gravity of four quadrants of a paper, where quadrant 1-left upper quadrant, quadrant 2, quadrant 3 to quadrant 4, clockwise, respectively. This agrees with our hypothesis. The most efficient position of magnets contains the cg, is symmetric, and exists between cg and CG(i)., It is very safe and efficient for individuals to hang up to 12 papers near CG(i) with 4 magnets. When considering the basic variable consisting of a rectangle, dist2CG and dist3CG were on the right side of the rectangle formed by the four magnets **(Table 1)**. This agrees with previous literature (4, 9).

We also performed prediction and prediction error (e.g., RMSE) analysis. The measured maximum value of papers held through experiment was 14 at this coordinate and the experimental maximum value of the real model has three cases was 14 at the CG(i) and near the CG(i). Although the best regression model has the highest adjusted R^2 value (0.926), the predictive error tends to be somewhat large at extreme values of 2 or 14 papers. For another robustness check, we also performed machine learning based analyses. The MLP with two hidden layers method was the most accurate, reducing the RMSE of 0.9165 paper compared with the BLM of 0.9259. As for feature importance, the ranking is quite similar to that of the regression analysis **(Figure 4)**. Overall, the difference between the two models is negligible. The CART result is a bit different from those of the BLM and MLP. The RMSE is very big and is 1.581 papers. The importance ranking of the most and second-most important feature are all the same for three models. And thus, our BLM is robust. In conclusion, it is most efficient to position the magnets near CG(i) considering efficiency, safety, aesthetics, and facility of comprehension.

The limitations of this study include identifying mechanical properties through experiments. Since numerous variables must be considered, it is difficult to control virtually all

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variables and conditions (10). For example, theoretically our dependent variable is a continuous one. However due to the heavy weight of a paper the result seems categorical. We have three cases with the same maximum number of papers. If we have much lighter paper then we may have better resolution of our results (e.g., one maximum case) and improve the model. Secondly, there was variability in the placement of the magnets which may lead to unsymmetrical placement of the magnets via trembling hand problem. The coordinate of the rectangle formed by the four magnets can be arbitrary since the magnets were positioned with human hands. Lastly, this study only considers the case of papers. If we use a thin tin sheet, the effects of bending at four corners would have been negligible.

This study successfully demonstrated the importance of the positioning of magnets for securing materials on vertical surfaces. This finding offers significant versatility. Our methods were creative and unique. We conducted experiments with minimal domestic equipment and used our data to analyze in a dry lab. Our innovative hybrid method is helpful for researchers under research budget constraints. Finally, with further testing and adjustment, our findings could be applied to mounting other, heavier materials on walls. In other words, the locations of the magnets are important. So, the points that produce 14 papers are the most efficient points in an economical sense. In summary, the locations of the four magnets with maximum efficiency are near CG(i).

MATERIALS AND METHODS

Experimental Design

We prepared blank A4 papers (4.7–5g) and selected 50 different positions in which we placed 4 circle-type magnets with a radius of 2mm in the shape of a rectangle **(Figure 1)**. The possible set of positions of four magnets were ${}_{461}P_4$ = 4617*4616*4615*4614. It was not possible to try all of the positions. We initially tried many different polygons with various numbers of magnets. Finally, we decided to experiment with only rectangular polygons using four magnets in up to 50 cases. The number of blank A4 papers between the surface of the refrigerator and the magnets in each position was increased by 1, and the maximum number of blank A4 papers each position could hold without slipping was recorded.

Because all data points in this study are rectangles and squares, this study's primary independent variables are only two magnets on the diagonal. However, it is not possible to represent a position in a variable. We need at least two variables such as distance from the reference point and an angle, or x and y coordinates. In our model we used only distance variables. However, there are infinitely many magnets on a circle with radius (distance) r. In order to identify a rectangle or a square, we need two reference points (e.g., cg and 4 CG(i)) and three distance variables. The cg is the center of gravity of a whole paper, while the four CGs, from CG(1) to CG(4), are the centers of gravity of four quadrants of a whole paper, respectively. Thus, our basic independent variables are at most 8 distance variables such as 4 disticg and 4 distiCG **(Appendix Table 1)**.

The remaining 20 variables reflect other reference point (4 corners) **(Appendix Table 1)**. Other tot_cg variables are the sum of the basic variables' values, and the tot_cg^2 variable is the square of the tot_cg variable, which measures the interaction or covariance of four basic variables. Finally, tot_sum_cgsq is the sum of squared of the values of each basic variable (variance in statistics and in this analysis, it is a degree of asymmetry with respect to the cg). After creating and inputting variables, we chose the best multiple linear regression model (BLM) based on the highest adjusted R^2 value **(Table 1 and Figure 6)**.

Coordinate System

We created our own Coordinate system; (x, 1 to 81) (y, 1 to 57) **(Figure 1)**. First, we determined the location of five important centers of gravity; One cg and four CGs. The cg is the center of gravity of papers while the four CGs, from CG(1) to CG(4), are the centers of gravity of four quadrants of a paper, where quadrant 1-left upper quadrant, quadrant 2, quadrant 3 to quadrant 4, clockwise, respectively. Our coordinate value is not a point, but a rectangle with area. Therefore, the coordinate value is not accurate but an approximation.

The coordinates of the cg were represented by (x_cg, y_cg) = (41, 29). The coordinates of CG(1) were (x_CG1, y_CG1) = (20, 44). The coordinates of CG(2) were (x_CG2, y_CG2) = (62, 44). The coordinates of CG(3) were (x_CG3, y_CG3) = (62, 14). The coordinates of CG(4) were (x_CG4, y_CG4) = (20, 14).

We then assigned names to the magnets holding the real paper. Regardless of the shape that the rectangle takes, the most left and upper magnet is called magnet 1 **(Figure 1)**. The others are numbered in clockwise order. Magnet 1 represents the most left and most upper magnet and the coordinate of magnet 1 is (x_1, y_1). Magnet 2 is the most right and most upper position and the coordinate of magnet 2 is (x_2, y_2). Magnet 3 is the most right and most bottom position and the coordinate of magnet 3 is (x_3, y_3). Magnet 4 is the most left and most bottom position and the coordinate of magnet 4 is (x_4, y_4).

Variables and Models

We have diverse distance measures based on multiple reference points (e.g., cg, four CGs and four corners of a paper). To determine which variable best predicts the outcome, we used a regression analysis. We ranked comparisons of the predictive power of the alternative regression models in terms of the coefficient of determination (adjusted R squared, R^2). Adjusted R^2 is a metric that tells us the proportion of the variance in the dependent variable of a regression model that can be explained by the independent variables. The higher the R^2 value, the better a model fits a dataset (11). RMSE is a metric that tells us how far apart the predicted values are from the observed values in a dataset, on average. R^2 and RMSE are correlated in linear regression models (11). First, we selected the testable hypotheses through previous literature review. Then, the model's predictive power can be compared with experimental results referred to in our study.

We used IBM SPSS 24 for all the estimation analyses. We also compared our BLM with log transformed alternatives. We used BLM as a benchmark linear-linear model. Log-linear model involves log-transforming the dependent variable only, while LOG-LOG involves transforming both the dependent and independent variables.

Before getting into further analysis, we needed to check the robustness of our results with respect to both overfit and multicollinearity. There might be an overfit problem considering our sample of 50 observations with 28 variables. Our BLM has only five input variables. Among them, two are

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second moments while the rest three are first moments with almost no correlation with each other **(Table 1)**. Thus, there is no serious overfit problem.

We input all 28 independent variables into the model, but multicollinearity problems occurred, so the model estimation result was not included in the manuscript. Using derivative variables such as tot_cg, tot_cg^2, and tot_sum_cgsq does not cause problems with our linear model, and it also mitigates problems such as omitted variable problem (1).

Regarding the feature importance ranking of each variable, we used an MLP and a CART method of IBM SPSS 24. For finding the maximum point via trial-and-error method from our estimated regression model we used Microsoft Excel, and for the visualization of the results we used MATLAB 2021. The meaning of the trial-and-error method reflects that not all positions of Magnet 2 on the A4 paper were targeted, and some positions outside the A4 paper were targeted to improve the granularity of model prediction. Briefly, it reflects the fact that it is not an analytical solution. Therefore, the estimated value of the results of this study may be different from the global maximum.

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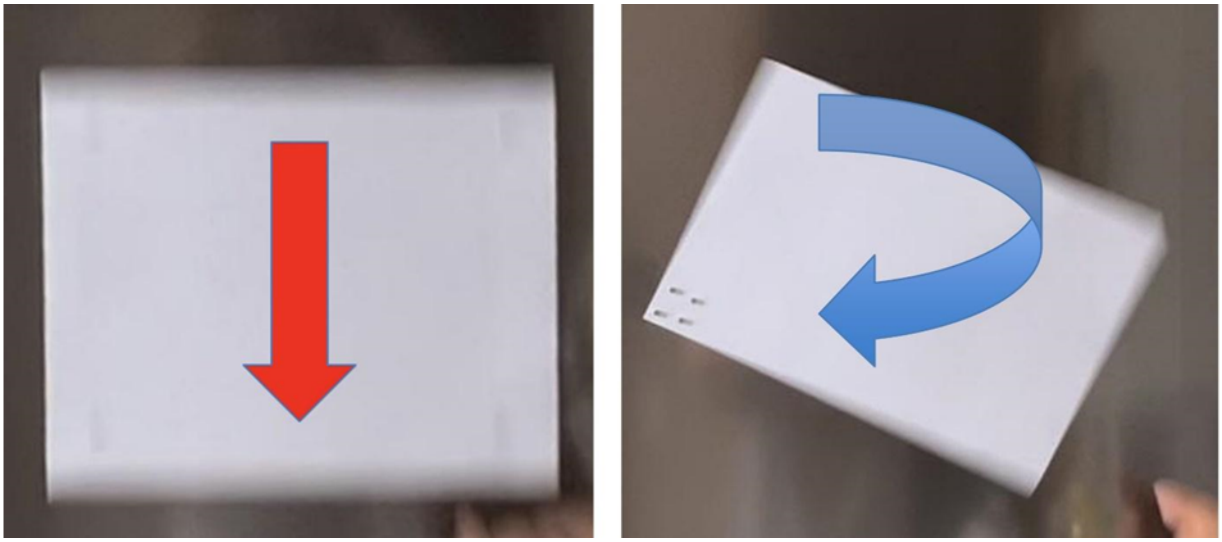
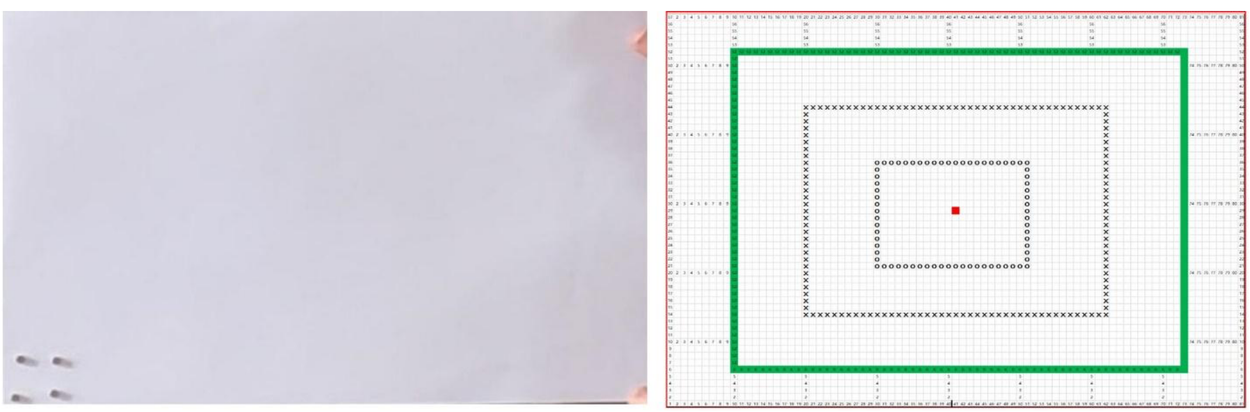
Appendix

Panel A. The action of gravity load Panel B. The action of torque

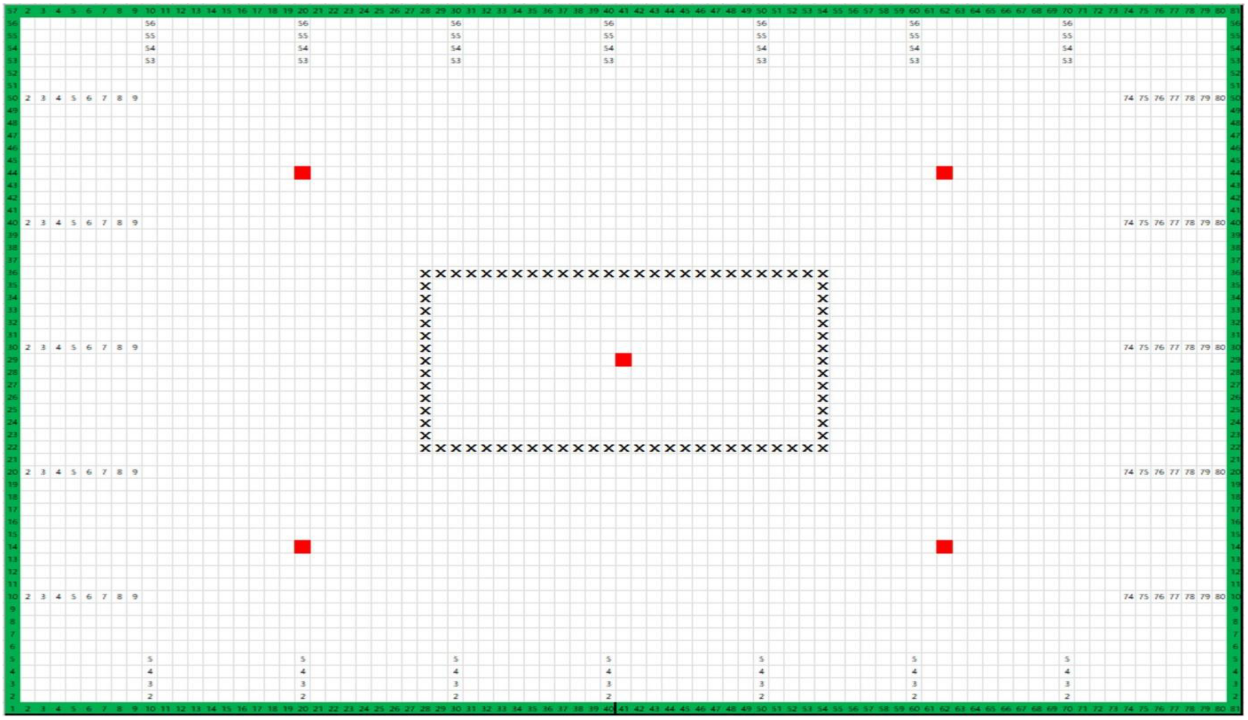


App.Figure 1. The actions of gravity load and torque

Panel A. minimum (2 papers) Panel B. Maxima (14 papers)



App.Figure 2. The experimental minimum and 3 maxima. The o and x marks and solid green line from inside out. The small red rectangle is the center of gravity of the paper.



App.Figure 3. The predicted max (13.2 papers) and trial-and-error max (12.6 papers). Small red rectangles: Center of gravity of the whole paper and 4 centers of gravity of the 4 quadrants outside. X-marked rectangle inside represents a trial-and error max. Green-rectangle represents a predicted max (13.2 papers) with actual value of 13 papers.

Variables	Interpretation	Calculation
tot_CG	total sum of distances of 4 magnets from the 4 quadrants' centers of gravity of the paper. sum of four $dist_i$CG(i), for i = 1,2,3,4	$dist_1$CG+$dist_2$CG+ $dist_3$CG+$dist_4$CG
tot_cg	total sum of distances of 4 magnets from the center of gravity of the paper. sum of four $dist_i$cg, for i = 1,2,3,4	$dist_1$cg+ $dist_2$cg+ $dist_3$cg+ $dist_4$cg
tot_CG^^{2}	squared value of the total sum of distances of 4 magnets from the 4 quadrants' centers of gravity of the paper	tot_CG*tot_CG
tot_cg^^{2}	squared value of the total sum of distances of 4 magnets from the center of gravity of the paper	tot_cg * tot_cg
$dist_i$CG	CG(i). distance between $magnet_i$ and CG(i), for i = 1,2,3,4.	sqrt((x_i- x_CG(i))^^{2} + (y_i- y_CG(i))^^{2})
$dist_i$cg	Cg. distance between $magnet_i$ and cg, for i = 1,2,3,4.	sqrt((x_i - x_cg)^^{2} + (y_i- y_cg)^^{2})
$dist_i$corner	$corner_i$. distance between $magnet_i$ and corner(i), for i = 1,2,3,4	sqrt((x_i- x_corner(i))^^{2} + (y_i- y_corner(i))^^{2})
Tot_corner	total sum of distances of 4 magnets from the 4 corners of the paper. sum of four $dist_i$corner, for i = 1,2,3,4	$dist_1$corner+$dist_2$corner+ $dist_3$corner+$dist_4$corner
Tot_corner^^{2}	squared value of the total sum of distances of 4 magnets from the 4 corners of the paper	tot_corner * tot_corner
Tot_sum_cornersq	total sum of squared distances of 4 magnets from the 4 corners of gravity of the paper	$dist_1$CG^^{2}+$dist_2$CG^^{2} + $dist_3$CG^^{2}+$dist_4$CG^^{2}

tot_sum_cgsq	total sum of squared distances of 4 magnets from the center of gravity of the paper	$dist_1CG^{\wedge 2}+dist_2CG^{\wedge 2}$ + $dist_3CG^{\wedge 2}+dist_4CG^{\wedge 2}$
tot_sum_CGSQ	total sum of squared distances of 4 magnets from the 4 quadrants' centers of gravity of the paper	$dist_1CG^{\wedge 2}+dist_2CG^{\wedge 2}$ + $dist_3CG^{\wedge 2}+dist_4CG^{\wedge 2}$
ln_paper	log transformation of paper variable	

App.Table 1. Variables description