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Article

The juxtaposition of anatomy and physics in the eye

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SUMMARY

People are quick to accept the assumption that a light will appear dimmer the farther away they are, citing the inverse square relationship that illuminance obeys as rationale. As a result, there are no studies that actually intend to verify this argument, given that the reasoning appears quite solid. However, repeated observations of light sources maintaining their brightness over large distances prompted us to explore how the brightness, or perceived illuminance of a light varies with the viewing distance from the object. When examining the anatomy of the eye, we found that the image it produces also decreases according to the inverse square of the viewing distance. Thus, we hypothesized that since both the illuminance of the light source and image size decrease at the same rate, then the concentration, or intensity of the image remains unchanged, and subsequently the perceived illuminance. To test our hypothesis, a circular light source was placed in line with a biconvex lens and light sensor, and the illuminance of the resulting image was measured at distances between 2–17 meters. We found that the illuminance of the image of light remained constant for most of the distances. Furthermore, the illuminance of the image increased at first before approaching that stable maximum. From our results, we concluded that our hypothesis was correct, and the decreasing image size indeed neutralizes the decreasing illuminance of the source. Beyond challenging previously held beliefs, our findings also have some niche aesthetic and economic implications for light fixture manufacturers.

INTRODUCTION

It is generally believed that the closer a viewer is to a light source, the brighter that source appears to the viewer, and vice versa. This belief is often rationalized by the fact that the illuminance of a light source decreases according to the inverse square of distance between the source and the eye (1).

We grew skeptical of this belief, however, as we observed multiple instances of light sources maintaining the same perceived illuminance—the number of lumens comprising a retinal image divided by the area of the image—over long distances. For instance, when driving at night along a long road with streetlamps, the lights at the far end of the road may appear smaller, but they all appear to be the same in illuminance. Similarly, the headlights of all opposing cars at night over a long highway stretch appear the same illuminance to our eyes. Furthermore, when flying over a city at night, the city lights towards the far end of the city look just as bright as the lights straight down below, even though the city may span several kilometers.

These observations are not represented in research. Most of the published research work regarding the perceived illuminance of lights is related to the brightness of stars, for which a stellar magnitude scale called the Apparent Magnitude scale is used to gauge their brightness with distances (2). However, the sheer distance of stars from earth renders them outside of the scope of our observations. Furthermore, these studies do not specifically examine the relationship between the perceived illuminance of a light and viewing distance, and thus do not offer explanations as to why various lights seem to remain the same perceived illuminance over large distances.

The scarce quantity of studies examining the aforementioned relationship may be explained by a number of reasons, one being the subject's interdisciplinary nature between the physiology and physics of human anatomy and lights. It may also be that the well-established knowledge regarding stars, which is consistent with intuition, is extrapolated to the light sources commonly found around us. Or it could be that the alternative to the common belief is incredibly counterintuitive, causing people to take the accepted rationale for granted.

Whatever the reason may be, we set out to explore this topic and define a relationship between the perceived illuminance of a typical light source and the viewing distance from the light. This perceived illuminance is not to be confused with the illuminance of the light source, which is already known to decrease according to the inverse square of the viewing distance (1). Rather, it refers to the illuminance of the retinal image. We conducted our research under the assumption that if the illuminance for two identical retinal images is the same, then the eye cannot distinguish any difference in brightness even though the size of the retinal image may be different.

We first considered the anatomic structure of the human eye (**Figure 1**). Though intricate, the eye is an image-forming optical device in which a number of refractive mediums focus the incoming light into a clear image (1). The retina in the back receives these images and the iris regulates the diameter of the aperture-like pupil, determining the amount of light that eventually forms the image (3). The light comprising this image stimulates the photoreceptor cells on the retina, producing the visual sensation of brightness, which varies with the illuminance of the image (4).

From there, we used the thin-lens and magnification equations to deduce that the size of the retinal image formed also decreases according to the inverse square of the viewing distance. Thus, we hypothesized: The decreasing rate of the image area effectively neutralizes the decreasing rate

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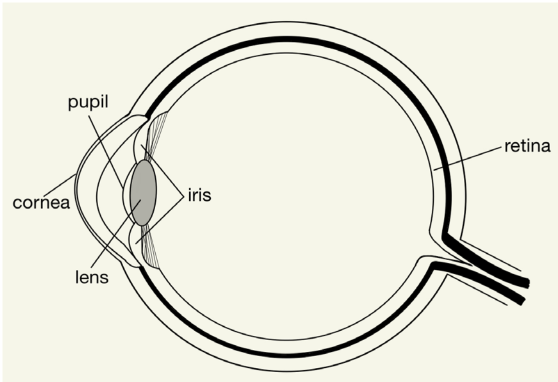

Figure 1: Anatomy of a human eyeball. A schematic diagram of the human eye with its main components essential to the contents of this research labeled.

of the illuminance entering the eye, as they are the same. Furthermore, since we assume that the eye cannot distinguish differences in brightness between two retinal images with the same illuminance, this offsetting effect renders the perceived illuminance of a light invariant.

Our results showed that the illuminance of the light remained stable over a major portion of the measurement distances. Surprisingly, we also found that before reaching a stable magnitude, the illuminance actually increased.

RESULTS

For experimental purposes, we simplified the complex mechanisms of the eye into its lens and retina, replacing the retina with a light sensor. We placed the light sensor behind the biconvex lens and measured the brightness at the sharpest image for viewing distances between 2–17 meters, moving in increments of 1 meter.

The illuminance remained at a stable level of approximately 194 lx for distances larger than 6 meters (**Table 1, Figure 2**). Additionally, the illuminance initially increased with increasing distance. Specifically, from 2 to 6 meters, the brightness of the image gradually increased from 151.8 lx to 194.7 lx,

Distance (m)	T1 (lx ± 0.5)	T2 (lx ± 0.5)	T3 (lx ± 0.5)	T4 (lx ± 0.5)	T5 (lx ± 0.5)	average (lx ± 0.5)	STDEV (lx)
2	151.7	151.8	151.7	151.8	151.8	151.8	0.1
3	165.1	165.1	165.2	165.5	165.4	165.3	0.2
4	174.0	173.9	173.9	174.1	174.0	174.0	0.1
5	182.2	181.9	182.0	182.2	182.2	182.1	0.1
6	186.5	186.1	186.2	186.1	186.2	186.2	0.2
7	188.6	188.6	188.4	188.4	188.4	188.5	0.1
8	194.7	194.9	194.9	194.7	194.5	194.7	0.2
9	195.6	195.4	195.5	195.2	195.3	195.4	0.1
10	194.8	194.7	194.7	194.6	194.9	194.7	0.1
11	194.6	194.5	194.6	194.6	194.6	194.6	0.1
12	194.1	194.0	194.8	194	194.8	194.3	0.4
13	193.9	193.7	194.0	193.9	193.8	193.9	0.1
14	194.3	194.4	194.3	194.2	194.3	194.3	0.1
15	192.4	192.5	192.3	192.5	192.5	192.4	0.1
16	192.9	192.3	192.4	193.4	192.4	192.7	0.4
17	195.2	194.8	194.8	194.4	194.4	194.7	0.3

Table 1: Illuminance (lux) measurements for viewing distances between 2-17 meters. Trial numbers are shown at the top of the table and the standard deviation (STDEV) for the measurements at individual distances is shown in the far-right column.

representing a considerable 28.26% rise (**Table 1, Figure 2**). The measurements for the illuminance fluctuated negligibly after the separation distance reached 8 meters, and always remained above 192 lx. The standard deviation for the measurements at all distances were very small indicating accurate measurement procedures (**Table 1**).

The initial rise in brightness for short distances allowed us to approximate the luminous flux of the light source we used. As shown by Eqn. 1, which we derived in the methods section, the $\frac{Ar^2}{4\pi f^2 R^2}$ term is effectively the slope of the brightness vs. $\frac{(D-f)^2}{D^2}$ graph (**Figure 3**). This graph is only useful since the illuminance increased at first. Had it remained constant the entire time, the slope of **Figure 3** would have been zero. For our experimental data, this slope came out to be 295 lx. This, along with the known values for r, R, and f, translates to a value of approximately 5,640 lm for the luminous flux of the source. This is a by-product of the brightness curve we derived, which could prove beneficial for light sources of unknown luminous flux.

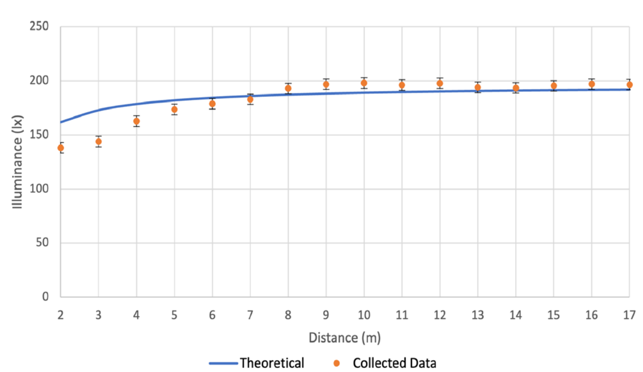

Figure 2: Experimental and theoretical illuminance vs. distance graph. The theoretical and experimental illuminance values of the light for viewing distances between 2–17 meters. Illuminance was measured by placing a light sensor behind a biconvex lens with a focal length of 0.185 m. The radius of the light source and lens used are 0.25 m and 0.0375 m, respectively. The error bars show the standard deviation of each measurement, which ranges from ± 0.04 lx to ± 0.42 lx.

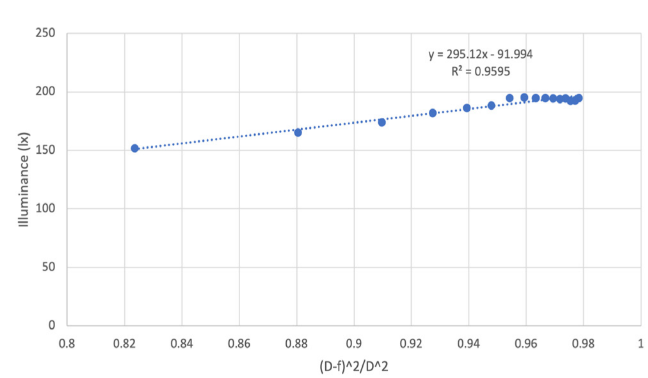

Figure 3: Experimental illuminance vs. $\frac{(D-f)^2}{D^2}$ graph. Trendline showing a slope of 295.12 lx and an R-squared value of 0.9595.

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Extremely far away objects seem to conflict with our experimental results as they appear to have a reduced perceived illuminance compared to nearby objects. We realized that objects eventually taper off in perceived illuminance and looked to diffraction for an explanation. More specifically, we analyzed how the perceived illuminance would behave as the image size approaches the size of the airy disk from diffraction.

The pupil is known to function as an aperture, diffracting incoming light. As a result, alternating bright and dark rings form around a central disk known as the airy disk as the lens focuses the light into an image on the retina (1). For larger images, the effect of diffraction is relatively unnoticeable, since the actual image size is much bigger. However, as the image size approaches the same size as this airy disk, the effect is much more prominent. More specifically, once the image size decreases below the size of the airy disk, the image gets blurred to approximately the size of the disk, effectively limiting the image to a minimum size (5).

Using an adapted form of Rayleigh's criterion (Eqn. 8) and the small angle approximation (Eqn. 9), we found that for a typical streetlight, 30 cm in radius, the image size reaches a minimum size of 0.77 arcminutes at a viewing distance of approximately 2.7 km (**Figure 4**). At this distance, the perceived illuminance of the streetlight would no longer maintain its magnitude. Rather, it begins to decrease according to the inverse square law (1).

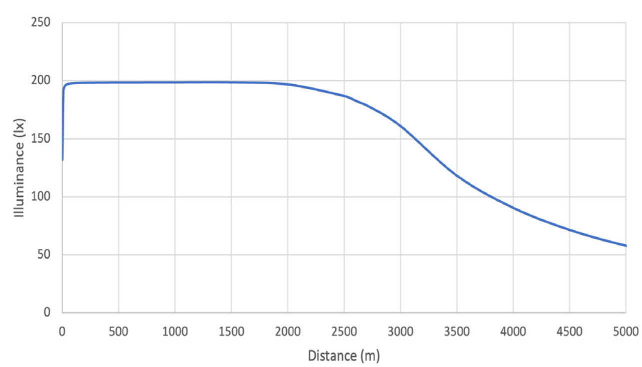

Figure 4: Expected illuminance values for all viewing distances of a streetlight. Scatter plot with smooth lines showing the expected illuminance drop-off distance for a streetlight 30 cm in radius. Threshold distance was calculated using Rayleigh's criterion as well as the small-angle approximation.

DISCUSSION

Our findings displayed a rather surprising profile for the perceived illuminance of a light—a plateaued curve with respect to distance. Although the perceived illuminance stays unchanged for a considerable range of distances, it initially rises before stabilizing at a constant magnitude.

The short end behavior of the curve can be explained by the $(D - f)^2$ term in Eqn. 1. When the distance is small, the focal length value is significant compared to the distance, causing the initial rise in perceived illuminance. We also visualized this trend by looking at the decrease rates of both the area of the image and the illuminance of the source. More specifically, as the viewing distance initially increases, the illuminance is decreasing much faster than the image area, resulting in a dimmer image. As the distance begins to dominate the focal length, however, the illuminance approaches the same decrease rate as the image, explaining both the initial rise as well as the stabilization in brightness.

While it might seem unreasonable that a light would appear dimmer up close than from further away, this trend can be explained by the focal length of the eye. Since the focal length of the lens in the eye is much smaller than the lens used to perform the experiment, approximately 17 mm, this dimming effect can only be seen from extremely close distances—approximately less than 10 cm (6). In reality, people rarely approach this distance with light sources. If they did, however, they would be overwhelmed by the brightness of the light and would most likely not be able to discern any differences in brightness. So, this effect is minimized in the real world and is of little consequence.

Although we did not explore the way in which the brightness curve transitions to its declining state in this research, it merits further research. For instance, the curve could suddenly begin decreasing or could gradually decrease (**Figure 4**). What is known, however, is that the perceived illuminance asymptotically approaches 0, explaining why some far away objects are invisible to the naked eye. From Hecht, Shlaer, and Pirenne, it is estimated that a minimum of 5–14 photons over a retinal area of approximately 500 rods is required to produce a visual sensation (7). At this scotopic threshold, the human eye will barely be able to distinguish the presence of an object. Anything below that threshold and the object will vanish from perception entirely.

While we reasoned that the limitation of human visual acuity was a result of optical diffraction, there is an anatomic explanation as well, which researchers often adopt to explain the resolution limit of human vision. In this competing view, the limit results from the physical size of the photoreceptors comprising the retina. It is impossible to distinguish differences in retinal image sizes smaller than the area of a single photoreceptor cell. In the fovea, the cones are approximately 2.5 µm in diameter (8). Since the focal length of the eye is known, the small angle approximation can be used to estimate the angular width of a single photoreceptor. Assuming a focal length of 17 mm, the angular width is roughly 0.51 arcminutes. This is rather close to the 0.77 arcminutes we estimated using Rayleigh's criterion. Thus, even though the two values were calculated based on very different viewpoints, they are consistent in determining visual limit, and, in turn, the drop off distance for perceived illuminance in our study. This consistency may be indicative of optimized human biological functions in the physical world.

A possible source of error in the experiment can be seen in a theoretical model we used earlier to derive the relationship between the perceived illuminance and distance, where the light source is assumed to be a point source illuminating uniformly in all directions. In the actual experiment, however, we used a circular light source of 25 cm in radius instead of a point source so that the images formed would be large enough for comfortable measurement. With a source of such size, the light from the edges entered through the lens at an angle unlike that of a point source, effectively allowing less light to enter the lens (**Figure 5**). To quantify the effect that a source of this size would have on the perceived illuminance, the source can be considered as a collection of concentric rings, and the amount of light entering the lens can be integrated

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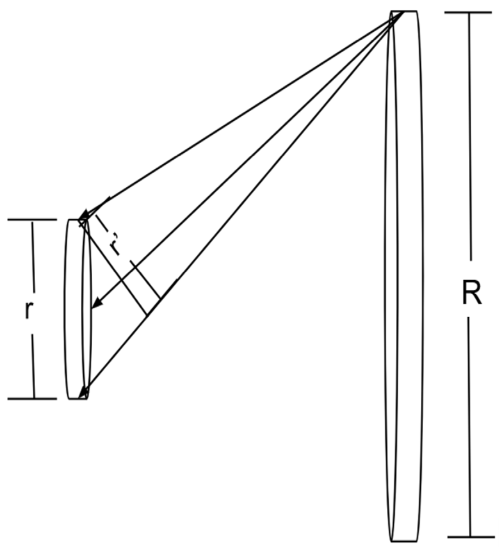

Figure 5: Visual representation of the resulting error from the use of a large light source. The light from the top of the light source enters an area equivalent to r' before entering the lens, which is a reduced amount.

across the radius of each ring. This results in an additional multiplier, $\frac{\ln(1+b^2)}{b^2}$, to the original illuminance expression (Eqn. 1) with b being the ratio of the source radius over the distance. Fortunately, it introduces negligible reduction in image illuminance for the distances in our measurements.

Additionally, the size of our basement limited our measurements up to a viewing distance of 17 meters. However, the focal length of the lens we used was sufficiently small, allowing us to capture a substantial amount of the stable portion of the perceived illuminance curve. Future considerations include conducting the experiment outdoors for more realistic surroundings and longer distances. With better equipment, future attempts should try to detect the drop off distance for the perceived illuminance of a light to verify our research.

Furthermore, our approximation of the image forming process of the human eye is simplified and does not mimic the eye's accommodation process. Since objects at different distances from the human eye will form images at different distances away from the lens, the eye must adjust the shape of its lens, or accommodate it, to make up for any discrepancies (1). Small muscles called ciliary muscles either contract or relax to alter the shape, and consequently the focal length of the lens, ensuring that the image formed lands directly on the retina, which is approximately 2.4 cm away from the outer edge of the cornea, the outermost refractive layer of the eye (1).

In our experiment, however, we used a lens of a fixed focal length. As the distance between the light source and the lens changed, so did the image distance—a variable image distance. Thus, the accuracy of our experiment is limited by our rudimentary approximation of the eye's image formation process. It should be noted, however, that at relatively large distances, the eye does not have to accommodate much, as the image is formed near the focal length of the lens. Therefore, the changing focal length has a negligible impact on the illuminance (**Figure 2**).

The findings of our research have several practical implications. The most apparent being that the perceived illuminance of a light source alone cannot be relied upon to determine the distance of the light. For instance, a lighthouse may be seen from the sea for up to several miles in the night. However, if the surrounding environment is pitch black, a boat in distress would have no idea how far it is from land.

Additionally, Eqn. 1 shows that a smaller source (i.e., a smaller R) translates to a smaller image, causing the lumens to unit area ratio to be larger. This means that manufacturers of decorative light fixtures can achieve a greater perceived illuminance without increasing the energy consumption simply by reducing the size of the fixture. Not only that, but they could save on both material and energy costs by proportionally reducing the size and energy consumption, all without suffering a reduction in perceived illuminance. The only drawbacks with decreasing the size or energy consumption would be aesthetics and that the threshold distance is decreased considerably. However, a reduction of the threshold distances would be negligible for decorative light fixtures, as they are not designed to be viewed from extreme distances anyway.

A possible extension of the experiment involves changing the color temperature of the light source, analyzing how a warmer source could affect not only the perceived illuminance, but the illuminance curve as a whole. For this experiment, the light source was set at 6,500 k, the highest possible setting. A condensed version of the experiment could be repeated, changing the temperature setting of the light each time. Even though the curve would have fewer data points overall as a result of condensing the experiment, the expected trend is already known from the experiment we conducted. This would be a particularly useful extension considering the varying color temperature of headlights, streetlights, city lights, etc.

Additionally, since our circumstances did not allow for the long-end behavior of the perceived illuminance curve to be experimented and tested, but rather estimated, further research is needed to fully explore both the transition of the curve, as well as the estimated threshold distance (**Figure 4**).

The experimental and theoretical results presented in this paper show a clear representation of the behavior of a light's perceived illuminance, which has not been previously investigated. Not only is a fundamentally important question answered, but the common misconception that the perceived illuminance of a light would change over the vast majority of viewing distances is also unambiguously dispelled.

MATERIALS AND METHODS

Experimental set-up

The experiment was conducted in our basement that had minimal natural light from the outside. A long length of blue tape spanning 17 meters was taped on the floor, which had distances in increments of 1 meter marked on it. The basement consisted of a large, rectangular main room, and a smaller storage room. The blue tape extended into the storage room from the main room of the basement. The storage room allowed for four extra meters of measurement while the door frame also blocked out the majority of the ambient light from the source.

The data collection station consisted of two nightstands placed in the back of the storage room (**Figure 6**). One of the

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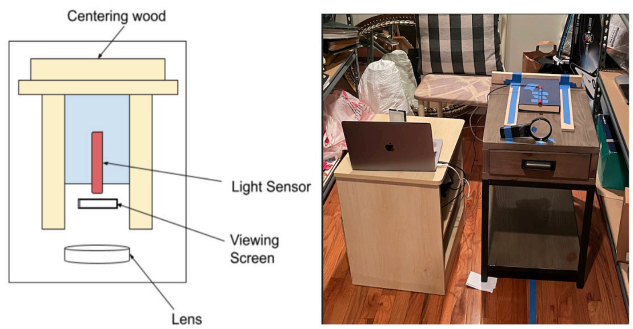

Figure 6: Data collection station. To achieve accurate measurements, the sharpest image was found with the viewing screen, then the centering wood holding the light sensor was pushed into the back of the screen. The screen was then removed, exposing the CCD of the sensor to the image formed by the lens.

stands held the recording computer (Apple, Cat. # A2338), which utilized LoggerPro to conduct the measurements. The other stand propped up a generic biconvex lens with a focal length of 185 mm and the light sensor (Vernier, Cat. # - LS-BTA). On the stand with the sensor, the lens was securely taped to the end of the stand closest to the light source (Nanlite, Cat. # FORZA60B-KIT), while the sensor was taped onto a 1.1-inch-thick book at the other end of the stand. The head of the sensor was taped slightly over the edge of the book, allowing it to be pushed directly into a viewing screen (Vernier, Cat. # OEK).

The light source was placed 17 meters away from the sensor to begin. At each distance, the viewing screen was placed behind the lens and in front of the sensor to locate the sharpest image. Small folds and imperfections in the gauze used to diffuse the light source were noticeable in the image on the viewing screen and were used to locate the sharpest image – the screen was moved back and forth until those folds were no longer blurry. After the sharpest image was found, a wooden contraption keeping the book parallel to the blue line on the floor was slowly pushed towards the viewing screen so that the head of the sensor contacted the backside of the screen, directly behind the image. The screen was then removed, allowing the image formed by the lens to shine directly onto the charge-coupled device (CCD) of the sensor, making sure the CCD was fully covered. Before recording any data, the viewing screen was re-introduced directly in front of the lens, which blocked all light from entering the lens. This process left only the ambient light, if any, to strike the sensor that would subsequently be zeroed out. The sensor collected data for approximately three seconds, after which an average was taken of the stable portion of the data—excluding any initial fluctuations resulting from the viewing screen. This marked one out of five trials for each meter mark. For the following four trials, the screen was not put back in front of the lens as the ambient light had already been zeroed out. This entire process was repeated for all distances between 2 and 17 meters.

The experiment was repeated three days in a row, with improvements being made after each day to stablize the collection procedure. These improvements included: switching from a lens with a focal length of 75 mm to one with 185 mm for larger images, the application of gauze over the light source to disperse uneven light distribution, the usage of a wooden contraption around the light sensor to keep the sensor parallel with the blue line on the floor, and the placement of the collection station inside of the storage room whose door frame blocked out a large portion of ambient light. Placing the station in the storage room also provided an extra four meters of measurement.

Derivation of Theoretical Expression

In the introduction, the perceived illuminance of an image was defined as the number of lumens striking the image divided by the area of the image. We reasoned that the number of lumens entering the pupil and subsequently the lens was also the number of lumens that comprised the image. Additionally, light was assumed to be a point source emitting light spherically. We stated that the ratio between the area of the lens, S, and the total surface area of the sphere of radius D was equivalent to the ratio of the number of lumens that entered the lens and the luminous flux of the source, L (Eqn. 1).

$$\frac{S}{4\pi D^2} = \frac{P}{L} \quad \text{(Eqn 1)}$$

S was replaced with πr^2 with r being the radius of the lens (Eqn. 2). Thus, P, the number of lumens entering the lens, and subsequently comprising the image, was

$$P = \frac{\pi r^2 L}{4\pi D^2} \quad \text{(Eqn 2)}$$

Finding the area of the image as a function of distance required the thin-lens equation (1) (Eqn. 3).

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{(Eqn 3)}$$

d_i is the distance between the image and the lens, f is the focal length of the lens, and d_o is the distance between the light source and lens, or D (**Figure 7**). d_o was replaced with D, and d_i was found (Eqn. 4):

$$d_i = \left(\frac{1}{f} - \frac{1}{D}\right)^{-1} \quad \text{(Eqn 4)}$$

d_i was then plugged into the magnification equation, $m = -\frac{d_i}{d_o}$ (Eqn. 5):

$$m = -\frac{f}{(D-f)} \quad \text{(Eqn 5)}$$

m was found in terms of D so that the radius of the image could be expressed as a function of distance by multiplying the radius of the source, R, by m (Eqn. 6).

$$A_{image} = \pi(mR)^2 = \frac{\pi R^2 f^2}{(D-f)^2} \quad \text{(Eqn 6)}$$

Once P and A_{image} were in terms of D, they were plugged back into the initial expression for perceived illuminance, $B = \frac{P}{A_{image}}$ and simplified (Eqn. 7).

$$B = \frac{Lr^2(D-f)^2}{4\pi R^2 f^2 D^2} \quad \text{(Eqn 7)}$$

This equation was written out in Excel and all variables were given fixed values except for D. For our experiment, the values of the variables used were as follows: r = 0.0375 m, f = 0.185 m, R = 0.25 m. L was unknown, but was solved for using the

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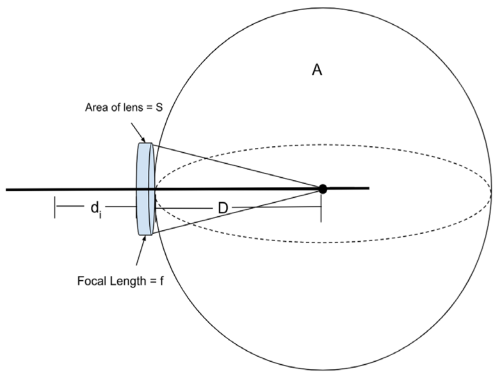

Figure 7: Model used to derive theoretical expression for illuminance at all viewing distances. The model shows a point source emitting light in a sphere with a radius (D) and surface area (A). A portion of the emitted light enters the lens with an area of lens (S) and a focal length (f), before being focused into an image at a distance (d_i) away from the lens. Not shown are the radii of the light source and image which are R and r, respectively.

slope of the graph in **Figure 4**.

Drop-off Calculations

From Rayleigh's criterion, $\theta_{rad} = \frac{1.22\lambda}{d}$, we knew that the size of the airy disk produced by the pupil was only dependent upon the wavelength of the light entering the pupil, λ, as well as the aperture diameter, d. However, the above form of Rayleigh's criterion is used to determine when the central maximum of one airy disk lies on the first minimum of another. Since we wanted to determine the angular width of a singular airy disk, we introduced a factor of 2 to the numerator (Eqn. 8).

$$\theta_{rad} = \frac{2.44\lambda}{d} \quad \text{(Eqn 8)}$$

At night, the pupil dilates to approximately 6 mm in diameter (9). The eye is also most sensitive to yellow-green light, which has a wavelength of approximately 550 nm (1). With these parameters, we found that the angular width of the airy disk formed by diffraction was approximately 0.00022367 radians, or 0.77 arcminutes.

Next, we used the small angle approximation (Eqn. 9) to estimate at what distance the image size reaches the angular width found above. The thin-lens equation (Eqn. 3) also works in this case, although the calculation is more involved.

$$\tan\frac{\theta_{rad}}{2} \sim \frac{\theta_{rad}}{2} \sim \frac{x}{L} \quad \text{(Eqn 9)}$$

x was the radius of the object whereas L was the distance between the object and aperture. To estimate at what distance the image of a streetlight becomes 0.77 arcminutes in angular width, we divided the object radius by the angular width in radians, then multiplied the quantity by 2. We found that for our example streetlight, the threshold, or drop-off distance is approximately 2.7 km.

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