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Article

Simulation of cosmic rays in the presence of a magnetic field

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SUMMARY

Cosmic rays damage microelectronics and have the potential to inflict great damage to life without the protection of the atmosphere and Earth's geomagnetic field (B field), which deflects the bulk of cosmic rays into space. When attempting to understand the observational measurements of cosmic rays, it is paramount to consider the effects magnetism will have on their trajectories. However, an analytical solution of the effects the B field has on cosmic ray trajectories is not possible because the cosmic ray flux is isotropic. We describe a new approach to numerically integrate, using Matrix Laboratory, the motion of cosmic particles deflected by a magnetic dipole field. We hypothesized that the numerical computation of cosmic ray trajectories through a magnetic dipole field, which approximates the Earth's B field, will validate the predictions of cosmic ray trajectories from known observational findings. The results of our study established the effects that the terrestrial magnetic field induces in the motions of these particles, as determined by the Lorentz Force. Overall, the trajectories of cosmic rays are determined by the particle's energy and its interaction with the Earth's B field. The high energy particles are curved, while the lower energy particles are either repelled or become trapped in the Earth's B field.

INTRODUCTION

Cosmic rays are high energy charged particles from space, the speed of which often approaches the speed of light. The term cosmic ray encompasses several different atomic and subatomic particles, such as electrons, positrons, protons, or other heavier atom nuclei; however, primary cosmic rays are comprised of protons. Protons are isotropic (same flux in all directions) to a very high degree. Upon impact with Earth's atmosphere, these rays produce a shower of secondary particles (1, 2). Each second, quintillions of cosmic rays strike the Earth indiscriminate of direction and orientation (3). The energy distribution of these particles ranges from the mega electron-volt (MeV) scale to enormously energetic events on the order of 10^9 Giga electron-volt (GeV) (4). Given their energy distribution, cosmic rays pose a significant risk to break through the protective shield provided by the B field and the atmosphere to damage avionics and living organisms (5,6). Further, there are studies which hypothesized possible climate changes because of the interplay between cosmic rays and the B field as well as the historical impact of this interplay on ancient civilizations through climatically driven environmental changes (7,8). Although the Sun is a source of emission of cosmic rays, the vast majority of cosmic rays are of interplanetary or intergalactic origin, emanating from supernovae, gamma ray bursts, quasars, and a myriad of other sources (9).

Previous studies have established that since cosmic rays are charged, their paths are modified by their interaction with magnetic fields (1,10). The principal force these charged particles experience in the presence of a magnetic field is known as the Lorentz Force (F), an electromagnetic force felt as a function of the particle's charge and velocity through an electric field E and magnetic field B. This Force is felt mutually perpendicular to both the direction of the velocity of the particle and field direction, and is given by the equation:

$$F = q(E + v \times B) \quad \text{[Eqn 1]}$$

Where q is the particle's charge, v is its velocity, and E and B are the electric and magnetic field vectors that interact with the particle. In a uniform magnetic field, the Lorentz Force causes charged particles to propagate in a helical path around a guiding center (2). The B field however is not a uniform field, rather it approximates a magnetic dipole field, meaning it resembles the magnetic field produced by a single closed loop with electric current flowing through it (11). Though the mechanism through which the Earth's B field is created is only theorized by physicists, it is believed that the mechanism is similar to the principle through which a self-exciting dynamo would function. In a dynamo an electromotive force (ε) is established, if a charged particle moves with a velocity v through an inducing magnetic field B, where $\varepsilon = v \times B$. The total current density arising from this electromotive force creates an induced field. The dynamo will be self-exciting if the induced field is the inducing field and vice versa (12). The convection of electrically conducting fluids within Earth's metallic outer core acts as a self-exciting dynamo, turning the Earth into a planetary geodynamo (11).

The Earth is not a perfect dipole and observational findings and simulations support the view that the convection currents within the Earth are turbulent, as is the resulting magnetic field. The rotation of the Earth clearly has an effect on the B field, and the magnetic field strength varies depending on latitude and longitude (13). While the magnetic and geographic poles do almost coincide, the magnetic poles reverse themselves on average every $10^3 – 10^4$ years, with strong reduction in field strength (14). The magnetic field lines are also modified by the interaction with solar wind, causing compression of field lines facing the sun and elongation in Earth's shadow. Research has shown that the differences amongst these magnetic field lines are greater at further distances from the Earth's surface (15).

Low energy cosmic rays are captured by the B field in two toroidally shaped zones around the Earth, known as the

Van Allen Belts (2). The magnetosphere's interaction with the solar wind can cause the precipitation of these trapped charged particles towards the Earth, which in turn causes excitation and ionization of nitrogen and oxygen within the atmosphere. As a consequence of this, the excited particles emit visible photons, which we perceive as the Aurora (16). This magnetic trapping is one of two mechanisms that stops cosmic rays from striking the Earth, the other being deflection. The Earth's B field changes the trajectories of cosmic rays, where depending on the particles' energies, they are either curved or repelled by their interaction with the B field (17). The presence of the B field also impacts the intensity of cosmic rays striking the Earth from east to west, a phenomenon known as east-west asymmetry (4). Positive and negative charges follow different paths in a magnetic field. Since the observational measurements showed higher intensities of cosmic rays being found west, the primary cosmic rays were deduced to be positive (18).

The minimum energy required for a particle to overcome the B field without getting deflected, can be calculated from the Størmer Integral and this minimum value is the Størmer Threshold, given by the equation:

$$P_{min} = \frac{\mu_0}{4\pi}\frac{qM}{R^2} \qquad \text{[Eqn 2]}$$

Where $\mu 0$ is the vacuum permeability of free space, q is the particle's charge, M is the Earth's magnetic dipole moment, which may be defined as the maximum amount of torque caused by magnetic force on a dipole that arises per unit value of surrounding magnetic field in vacuum, and R is the radius of the Earth (19).

There are several types of modelling used to study the effects of the B field on cosmic ray trajectories. Data-driven models use observational data to infer the effects B field has on cosmic rays and can be used to validate or improve the accuracy of other types of models (20). Machine learning models, such as neural networks, are trained using observational data and can be used to make predictions of cosmic ray flux where data is not available (21).

Numerical simulations use different numerical techniques to compute the cosmic ray flux in Earth's atmosphere (22). Since the geomagnetic field deviates from a pure dipole, the accuracy of prediction depends on the accurate dynamic geomagnetic field models. In our analysis, we assumed a magnetic dipole field which approximates the Earth's B field but expect our methodology would work equally well for different B fields. Using simulations, we were able to accurately model the main observed features of cosmic rays, including the B field's shielding effect, the east-west symmetry of the particles and, Størmer cutoff rigidity.

RESULTS

We created computational simulation in Matrix Laboratory. A numerical computation provides a means of predicting the path of cosmic rays when an analytical solution to the problem is not possible. This is because cosmic rays are isotropic with uniform distribution in all directions, and this characteristic makes it difficult to predict the specific paths of individual cosmic rays as they are not preferentially directed towards or away from the B field.

Prior to simulating the motion of a charged particle in a magnetic dipole, a simulation was run with a uniform magnetic field. The resulting plot verifies that the particle behaves according to the expected outcome in this scenario, obeying the Lorentz Force law (**Figure 1**).

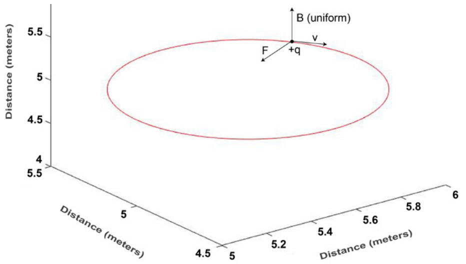

Figure 1. Motion of a particle due to Lorentz Force in a uniform magnetic field. In the simulation the uniform magnetic field acts through the z axis and the particle velocity on y axis, where the acceleration of the particle is mutually perpendicular to both the field direction and particle velocity. As such, the particle propagates around a guiding center, which is tangent to the direction of velocity.

We first attempted to model the shielding effects by which the B field deflects the positively charged protons each with energies of 26 GeV and with different starting positions (**Figure 2A)**. The proton particles 1, 2, and 3 are in the equatorial plane and their motion remains restricted to the

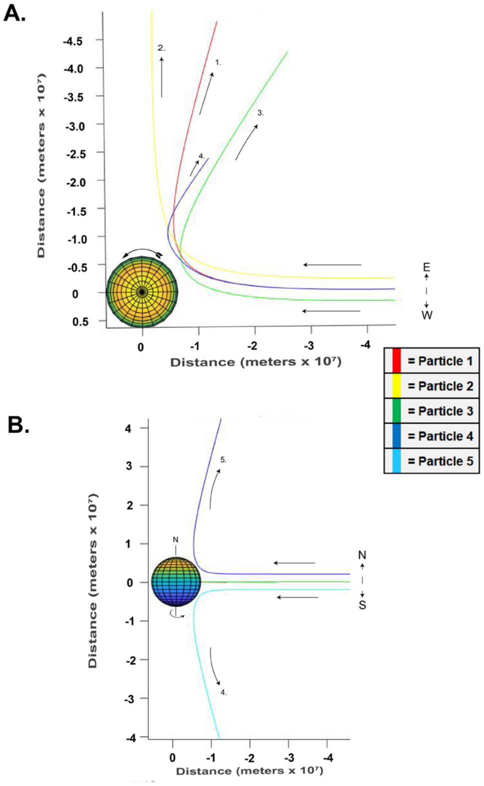

Figure 2. Deflection by the B field of positively charged protons (particles), with an energy of 26 GeV each and different initial spatial coordinates. A) Top down view of the Earth, where particle 1 approaches the Earth from the celestial equator, with zero ascension. particles 2 and 3 still exist on the equatorial plane but are displaced east and west 2,000 kilometers respectively relative to particle 1. **B)** Vertical perspective projection of the Earth, where particles 4 and 5 are displaced 2,000 kilometers north and south of the equatorial plane. The simulation illustrates the deflection of positively charged protons (particles) from different initial spatial coordinates.

plane. This is because the acceleration of the particle is the cross product between the particle's velocity and the B field. The resulting vector for acceleration in this scenario means that the z component of the particle's velocity remains zero, and the positively charged particles curve toward the east. Since the particles 4 and 5 are displaced north and south of the equatorial plane respectively, they are deflected north and south, as well as east **(Figure 2B).** This simulation clearly illustrates the shielding effect by which the B field screens the planet from most cosmic rays aimed at it.

Next, we wanted to test how the direction of the particle's velocity impacts its trajectory when present in the B field (**Figure 3**). The trajectories of particles 2 and 3 demonstrate the east-west effect of cosmic ray intensity distributions. The positive charges of the particles entail that particle 3 curves such that it strikes the Earth, while particle 2 curves such that it avoids the Earth.

We also tested the trajectories of charged particles with different energies to test the minimum energy required for a particle to overcome the B field (**Figure 4A**). The minimum energy for a particle travelling along the equatorial plane to overcome B field is calculated from the Størmer Integral using Equation 2 above, where the assumed value in the simulation for the magnetic dipole moment of the Earth (M) was 7.79 x 10^{22} ampere meters squared, and the resulting value obtained from Equation 2 for a proton is 57 GeV (23). The simulation was repeated for particles with energies progressively increasing from 1.4 GeV to value closer to the Størmer Threshold to confirm that these particles will not strike the Earth. The repeat simulations of two particles of 60 GeV (Red) and 50 GeV (Green) are shown where the circular outline represents the section of the Earth sliced along the equatorial plane **(Figure 4B).** The only particle to reach the Earth in these simulations is the particle with energy of 60 GeV, which is above the predicted Størmer Threshold.

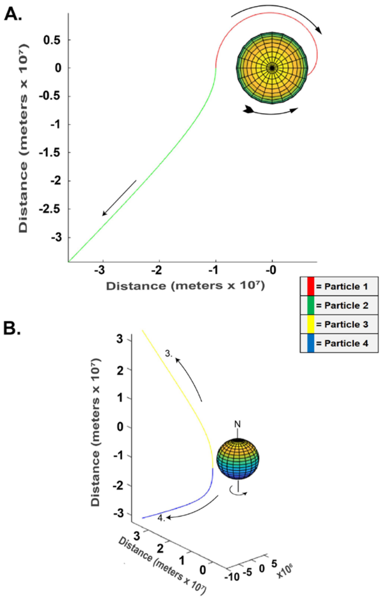

Figure 3. Trajectory of the particles with different direction of velocity, the positively charged particles show an East-West asymmetry. Trajectories of particles with different direction of velocity, where the B field impacts cosmic rays intensity from east to west. All particles start in the same position at the equatorial plane. **A)** Trajectories of particles 1 and 2 with east and west oriented velocities. **B)** Trajectories of particles 3 and 4 with velocities oriented to the north, and south, respectively.

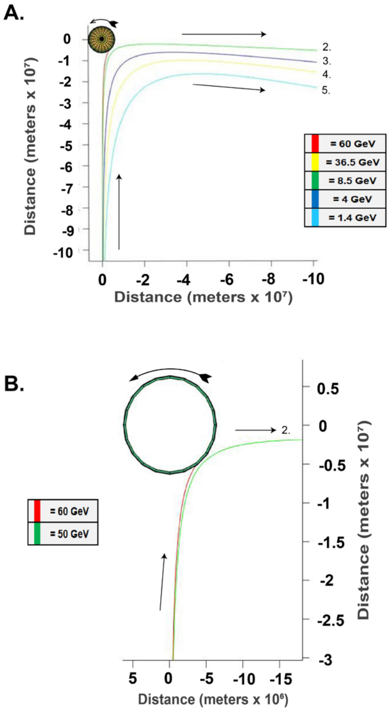

Figure 4. Trajectories of cosmic ray particles travelling along the equatorial plane. A) Simulation showing the trajectories of particles having different kinetic energies to determine which particles reach the Earth, where all particles start in the same position, 190,000 km from the center of the Earth. **B)** Cross-sectional view of the Earth showing repeated simulation with energies above and below the Størmer Threshold validating the energy required to hit the Earth. The circular outline represents the Earth sliced along the equatorial plane.

We also modelled the ability of the B field to act as a barrier that holds high energy particles within itself. The simulated trajectories of proton, shows that it gyrates along a magnetic field line of the Earth. Once the rotation of the particle reaches a pitch angle equal to 90°, it begins to gyrate in the opposite direction towards its magnetic conjugate point **(Figure 5).** The proton magnetically trapped in the simulated B field was akin to the magnetic trappings around the Earth in the toroidally shaped zone known as the Van Allen Belts.

DISCUSSION

Conceptually, this simulation is approximating the Earth's geomagnetic field and studying how it deflects the cosmic particles. It simulates the movement of the particles over time and how it is affected by the magnetic field and the Lorentz force. The result is a representation of the path of the particles in space, and how the magnetic field has affected their movement. The simulation developed in the present research validates our hypothesis as it reproduces all the main observative features of the flux of cosmic rays onto the Earth: the overall shielding effect of the planetary magnetic field, the quantitative threshold on the energy of observed particles, their east-west asymmetry, and even more complex phenomena such as magnetic trapping in the Van Allen belts

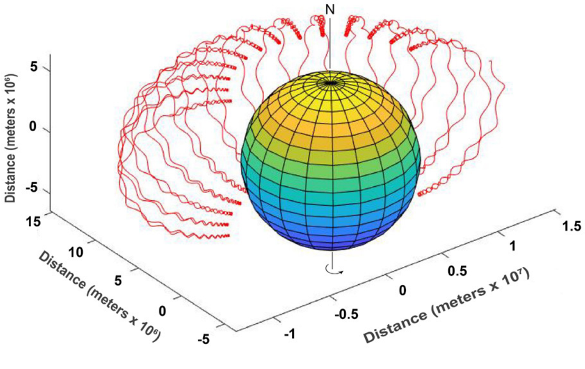

Figure 5. Protons magnetically trapped in a toroidally shaped zone by the Earth's magnetosphere. The magnetic trappings with simulated trajectories of proton with energy of 240 MeV and travelling at 60% the speed of light.

and the formation of the Aurora Borealis and Australis. Our simulations establish that the path of charged cosmic rays are modified in the presence of the Lorentz Force exerted by the B field, which approximates a magnetic dipole. Our data also provides computational support for the value of the Størmer Threshold, as only a particle with energy of 60 GeV can overcome the B field and strike the Earth since it has a value above the Størmer Threshold. The nature of the B field and the elongation of the field due to solar wind would impact the observational measurements.

Positively charged particles when approaching the Earth in the equatorial plane are deflected east, while those not in the equatorial plane are deflected not only east, but also to the northern or southern hemispheres depending on how they are approaching the Earth. The presence of the B field precludes certain cosmic ray trajectories; positive and negative charges follow different paths in the presence of a magnetic field such that positively charged cosmic rays are able to curve and strike the Earth when approaching from the west and avoid the Earth when approaching from the east. This supports observational measurements of higher intensities of cosmic rays being found westward (18).

Computation modelling shows that protons would be magnetically trapped, in a toroidally shaped Van Allen Belt, when their energy ranges from 100 Kilo electron-volt (keV) to several hundred mega electron-volt (MeV). For electrons, this number is between 10 keV and 10 MeV, subject to the limitations of the presumed B field. The trapped particles also go through a process of drift, where the particle circles around the Earth. Positive charges, such as the particle shown in the simulation, drift westward. If the particle was instead an antiproton or electron and had a negative charge, it would drift eastward (2).

There are some limitations of the simulation model. The Earth's magnetic field is not a perfect dipole. The tilt of the Earth's axis and the dynamical forces within the Earth's core cause magnetic anomalies and the B field fluctuates in the near-Earth environment. This has not been factored into the simulation and the assumed B field is an approximated dipole which could impact the accuracy of the results. Further the elongation of the field lines due to solar wind could considerably impact the observational measurements and the simulation does not take this into account.

The outcome of this research is significant because due to the computational nature of this study it can easily be extended and applied to enhance our understanding in other areas of research. The effect of variations of the B field due to the solar activity or to the inversion of magnetic poles, and in assessing other hypothesis, such as possible climate changes because of the interplay between cosmic rays and the B field. For example, to estimate the likely increase in geomagnetic cutoff rigidity (with reduction in B field following geomagnetic reversal) and its impact on atmospheric cloud nucleation and warmer temperatures (24).

MATERIALS AND METHODS

To numerically compute the trajectory of the charged particles through a magnetic dipole field, which approximates the B field, the B field strength at the position of the particle was calculated separately for each of the particle's axes that the field acts upon. The values for the particle's x and y axes are calculated independent of each other, but each with respect to the z axis. The magnetic field for the x and y directions and the z axis, are accordingly given by the equations:

$$B_x = \mu_0 \frac{3xz}{r^5} \quad \text{[Eqn 3]}$$

$$B_y = \mu_0 \frac{3yz}{r^5} \quad \text{[Eqn 4]}$$

$$B_z = \mu_0 \frac{2z^2 - x^2 - y^2}{r^5} \quad \text{[Eqn 5]}$$

Where Bx, By and Bz are the magnetic field strength values, $\mu 0$ is the vacuum permeability of free space. The x, y and z are the particles position along their respective axis (where the center of the Earth lies at the origin), and r is the distance of the particle from the center of the Earth. These equations represent the magnetic field strength at a point, for a magnetic dipole field.

Relativistic Correction

The corresponding magnetic field strength values calculated from **Equations 3, 4, and 5** were then used to calculate the particles acceleration. In order to accurately ascertain the particles trajectory at velocity values close to the speed of light, a relativistic correction of the Lorentz Factor, given by following equation, was applied to the particle's acceleration:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{[Eqn 6]}$$

Where, γ is the Lorentz Factor, v is the particle's velocity and c is the speed of light.

Conceptually, the code models a particle with a charge and uses the Lorentz force equation to calculate the force acting on the particle as it moves through the magnetic field. The initial conditions for the cosmic particles, such as their position, velocity, and mass are defined. The magnetic field of the dipole is presented by a vector field. The equations of motion for the cosmic particles are numerically integrated, updating the position, velocity at each time step, and the resulting particle trajectory is plotted in a 3D representation. The computational simulations were conducted in Matrix Laboratory (MATLAB). The code being open source can be further optimized and improved, for example by incorporating the effects of changes in the B field strength and direction over time and space. The simulation code has been written in MATLAB and is available at: https://github.com/prajorv1/Simulation-of-cosmic-rays-in-the-presence-of-a-magnetic-field/blob/main/main.

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