

An optimal pacing approach for track distance events

Kaitlyn Lee¹, Adrian Lee¹

¹ Sacred Heart-Griffin High School, Springfield, Illinois

SUMMARY

Track athletes aiming to compete at the highest level often must meet specific qualifying standards for middle and high school state championships, national collegiate championships, and ultimately, the Olympic Games. In this project, we aimed to address the best approach for achieving personal best times in track distance events. We used an existing mathematical model based on physiological attributes of world-record-setting elite runners to yield an optimal pacing approach. We then confirmed the validation of this energy depletion model using elite men's and women's gold medal performances at the Tokyo 2020 Olympic Games. We hypothesized that the average pace of a field of high school athletes competing in 800 m, 1600 m, and 3200 m distance events at a championship track meet does not follow the optimal pacing profile. Instead, we believed many runners begin with a fast pace to stay within close range of the race leaders, while others start conservatively to save energy for the final stage of the race. We used official timing data to test our hypothesis against computational simulations. We found that the average pace could deviate from the theoretical optimal pace by as much as 4%, translating to a difference of 1-2 seconds every 200 meters. Our analysis helps middle and high school athletes understand how pacing can improve their personal best times and what training can be performed to improve their physiological capabilities.

INTRODUCTION

The sport of running exists across a variety of recreational and competitive events. People of all ages and abilities gather throughout the year to run in charitable 5 km, 10 km, half marathon, and marathon races. At the sport's pinnacle, elite track athletes race against one another over distances ranging from 100–10,000 m in World Athletic competitions and the Olympic Games (1). Middle school, high school, and collegiate athletes compete in cross country in the fall, then transition to indoor and outdoor track in late winter and spring. Indoor and outdoor tracks of 200 m and 400 m, respectively, are the most convenient way to compare time performances across various venue locations. Weather conditions and track surface quality can both influence performance (2). However, the symmetric and level layout of a track is ideal for monitoring pacing variations over regular interval distances.

In competitive track meets, finishing placement is the valued prize. Race strategies often play a crucial role towards who ends up victorious. For example, individuals may

utilize a “sit and kick” approach by strategically positioning themselves within the field of runners, saving energy to surge past their competitors in the final stages (3). In this approach, techniques involving drafting or running in a pack have been shown to improve running economy – the energy used while running at a given pace – and allow trailing runners to gain a benefit in pace by up to 2.3% (4). Elite runners may also vary their pace and tactics based on specific zones on the track (5). In contrast to championship racing, however, many runners also revel in achieving personal records (PRs) or finishing in their fastest possible time (6).

In this study, we aimed to educate young athletes how to run 800 m to 3200 m distance track events in their fastest time. These events last longer than 120 s and have been found to be optimized by even pacing (7). Shorter distances covering < 80 s (up to 291 m) are shown to be run with maximum effort (8, 9). In elite 800 m runners, greater speeds are often achieved during the first lap combined with a slower second lap (7). Longer distances—such as road races and cross country—often involve variation of speed and effort over hilly terrain.

The performance of middle and high school track athletes covers a wide range of abilities. Many athletes have been running for several seasons beginning in their elementary school years. In contrast, several athletes may join track as a new sport. At an early regular season high school track meet, one can often witness an over-exuberant 400 m or 800 m runner run too fast over the first half of the race, then drop to a slower pace over the remaining distance. We decided to verify whether high school runners follow the optimal pacing approach by analyzing track results from a competitive meet involving top high school athletes.

We hypothesized that the average pace of high school track athletes competing in distance events does not follow the optimal pacing profile. Many try to keep up with the race leaders and run excessively fast in the beginning stages of the race. Unfortunately, this approach exceeds their capabilities and leads to an early onset of fatigue and suboptimal time performances (10). Using a set of energy depletion model equations, we generated a computer simulation to provide a fast and convenient method of comparing a theoretical optimal pacing approach to real-world timing data. This model was then compared to gold medal performances at the Tokyo 2020 Olympic Games. By comparing the optimal pacing approach to timing data available from high school track events, we found evidence supporting our hypothesis. We identified physiological parameters using a steady-state analysis to illustrate how high school runners can follow the optimal pacing approach to achieve new PRs. Lastly, we provide a discussion pertaining to the physiological attributes to help track athletes understand how specific training

	τ (s)	f_{max} (m/s ²)	σ_m (m ² /s ³)	e_0 (m ² /s ²)
Men's 800 m	0.836	10.07	22	5160
Women's 800 m	0.818	9.38	21	4546
Men's 1500 m	0.929	8.55	26	6519
Women's 1500 m	0.875	8.11	24	5726

Table 1: Simulation parameters for elite athletes competing in the Tokyo 2020 Olympics.

methods can improve aspects of their own personal ability levels.

RESULTS

Elite Athlete Performance Analysis

We used a mathematical model based on Newton's second law and the depletion of anaerobic energy in the muscles to simulate the performance of elite athletes competing in 800 m and 1500 m distance events. The goal of this analysis was to characterize the athlete's capabilities by determining a set of physiological parameters, then validate the model using the elite athlete's Olympic performance. We compared the optimal pacing profile against the official split times using mean and standard deviation statistics to understand the athlete's racing strategy.

Physiological parameters contained in the energy depletion model include the athlete's running economy, τ , the maximum propulsive force, f_{max} , the maximum oxygen uptake rate, σ_m , and the initial anaerobic energy stored in the muscles, e_0 (see Equations 1–4 in "Materials and Methods"). Numerical values for these parameters can be found in published literature based on elite men's 800 m and 1500 m world record times (10, 11). However, the authors did not obtain parameters for elite women's distance events. Due to the availability of official split times, we used gold medal performances set at the Tokyo 2020 Olympics to establish a new set of physiological parameters for both elite men and women in the 800 m and 1500 m events. To minimize the number of free parameters in the simulation model, we set the percentage of maximum applied propulsive force during the middle phase of the race at 90% for all events, $f_{middle}(t) = 0.9f_{max}$. Likewise, we maintained the anaerobic thresholds, $\gamma_s = \gamma_f = 0.15$, and oxygen uptake rates, $\sigma_s = 6$ and $\sigma_f = 0.9\sigma_m$, across all events. Then, we fine-tuned the four remaining parameters to produce theoretical split times with near identical finishing times to those in the official race results (Table 1).

Consider the men's and women's 1500m race, for example. We designed the runner's propulsive force profile to follow a sinusoidal curve over the starting and finishing phases of the race (Figure 1A). The stored anaerobic muscle energy decreased over the duration of the race, reaching 0 m²/s² at the finish (Figure 1B). The model simulation showed the runner to exhibit an initial surge in velocity over the beginning phase of the race, followed by a constant velocity over the middle phase, then conclude with a final velocity increase through the final phase (Figure 1C). The oxygen uptake rate transitioned through the three race phases as governed by its piecewise function in Equation 4 (Figure 1D).

Following the identification of the physiological parameters, simulation of the energy depletion model provided a comparison between the optimal pacing approach and the official split times for the women's and men's 800 m and 1500 m events (Figure 2). The mean percent differences

the athletes ran from the optimal pace profile were 3.8% (women's 800 m), 5.7% (men's 800 m), 3.9% (women's 1500 m), and 1.7% (men's 1500 m), with standard deviations of 2.2% (women's 800 m), 3.7% (men's 800 m), 3.5% (women's 1500 m), and 1.2% (men's 1500 m).

An additional benefit gained from the model simulation is the insight into the distance covered during each of the three phases of the race. For example, the optimal pacing strategy for the beginning phase of the race covered 101 m in the men's 800m event and 171 m in the men's 1500 m event (Table 2). The distance covered during the final phase of the race is slightly longer than that of the initial phase—112 m in the men's 800 m event and 205 m in the men's 1500 m event.

High School Athlete Performance Analysis

After validating the energy depletion model against the elite athlete performances, we used the model to understand how high school athletes approach competitive distance races. We characterized the physical attributes of girls and boys competing in 800 m, 1600 m, and 3200 m indoor track events. Then, we compared how the runners, on average, followed the optimal pacing profile using mean and standard deviation statistics.

First, we determined simulation parameters for each of the girls' and boys' events to yield finish times identical to

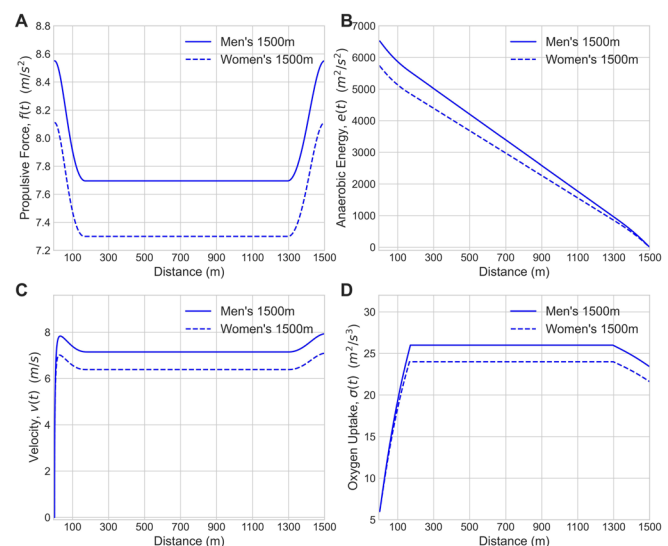


Figure 1: Mathematical simulation of the energy depletion model. Graph of the men's (solid line) and women's (dotted line) 1500 m Olympic records showing (A) the propulsive force profile, (B) the anaerobic muscle energy, (C) the optimal velocity profile, and (D) the oxygen uptake rate as a function of the distance traveled. A computer simulation of the energy depletion model equations was generated in Python with the set of physiological parameters fine-tuned to match the athlete's official finishing time.

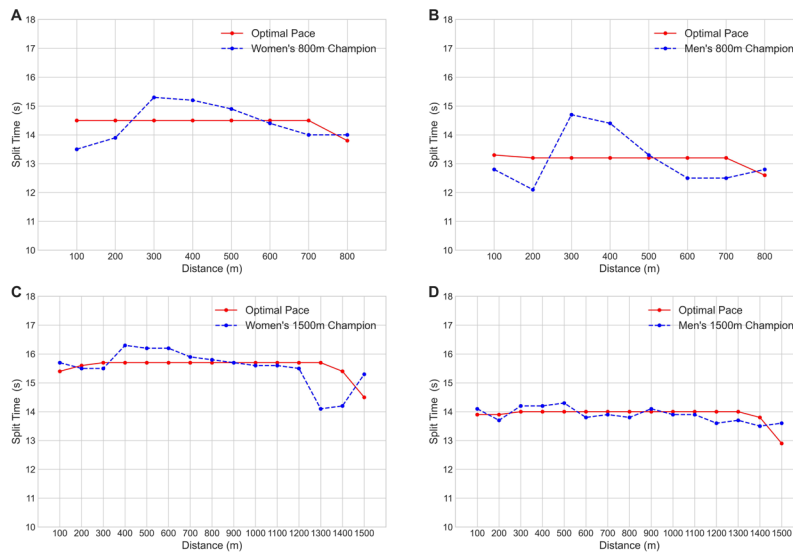


Figure 2: Comparison of elite athlete performance to optimal pacing approach. Graph of the official split times (dotted blue lines) for the (A) women’s 800 m, (B) men’s 800 m, (C) women’s 1500 m, and (D) men’s 1500 m gold medalists from the Tokyo 2020 Olympics are shown in comparison to the optimal split times (solid red line). Computer simulation of the energy depletion model was used to generate the optimal split times. Physiological model parameters were identified using a steady-state analysis and fine-tuned using the women’s and men’s Olympic finishing time.

	Men’s 800 m	Women’s 800 m	Men’s 1500 m	Women’s 1500 m
Start phase distance	101 m	99 m	171 m	166 m
Middle phase distance	587 m	590 m	1124 m	1132 m
Finishing phase distance	112 m	111 m	205 m	202 m

Table 2: Distance covered during the start, middle, and finishing phases of each Olympic event.

the field average finish times (see Methods and Materials) (Table 3). Using these parameters, simulated split times from the energy depletion model indicated that the field ran as much as 3 s faster than the optimal pacing approach during the middle phase of the girls’ and boys’ 800 m races, >5 s faster in the girls’ 1600 m race, >7 s faster in the girls’ 3200 m race, and >9 s faster in the boys’ 3200 m race (Figures 3A-C, 3E-F). Only the boys’ 1600 m competitors, on average, kept within 1s of the theoretical optimal split times (Figure 3D). The mean percent differences the average pace deviated from the optimal pacing profile were 3.9% (girls’ 800 m), 4.2% (boys’ 800 m), 3.1% (girls’ 1600 m), 1.3% (boys’ 1600 m), 2.3% (girls’ 3200 m), and 3.0% (boys’ 3200 m). With 200 m split times of 30.7 s for the boys’ 800 m, for example, this deviation translated to a difference of 1.29 s each lap. The standard deviations of the average pace to the optimal pacing profile were 1.5% (girls’ 800 m), 2.0% (boys’ 800 m), 2.1% (girls’ 1600 m), 0.5% (boys’ 1600 m), 1.7% (girls’ 3200 m), and 2.3% (boys’ 3200 m).

Simulation of the energy depletion model using these parameters again provided insight into the distance a runner covered within each phase of the race (Table 4). The distances covered by the boys during the start and finishing phases were slightly longer than the girls in each event. Notably, the higher speed start constituted approximately 10–13% of the event distance, while the finishing surge in pace covered approximately 13–14% of the event distance.

	τ (s)	f_{max} (m/s ²)	σ_m (m ² /s ³)	e_0 (m ² /s ²)
Boys’ 800 m	0.722	10.02	19	5108
Girls’ 800 m	0.630	9.33	18	4225
Boys’ 1600 m	0.736	8.61	21	6962
Girls’ 1600 m	0.665	8.08	20	5457
Boys’ 3200 m	0.766	7.54	23	8380
Girls’ 3200 m	0.673	7.10	21	5510

Table 3: Simulation parameters for high school athletes competing in the indoor championships.

DISCUSSION

This study utilizes an energy depletion model to mathematically simulate athletes competing in distance track events. The optimal solution provided insight into the level of effort a runner should apply during the beginning, middle, and end phases of the race. Our contributions to this area of research include (i) the creation of a sinusoidal propulsive force profile, (ii) insight into the distance covered during each of the three race phases, (iii) a novel method for estimating physiological parameters using steady-state analysis, and (iv) the adaptation of the energy depletion model to analyze middle and high school track performances.

The optimal solution of the energy depletion model (see Equations 1–4 in Materials and Methods) is solved as a calculus of variations problem (9, 12). The mathematical model consists of two differential equations and three unknown functions, $v(t)$, $e(t)$, and $f(t)$, while the constant parameters τ , f_{max} , σ_m , and e_0 are physiological attributes of the runner, which may be identified through experimental techniques (12). The optimal solution to the distance pacing problem incorporates a fast start with strong acceleration, followed by a constant velocity during the middle part of the race, and concludes with a final sprint at maximum force (11, 13). Indeed, studies show

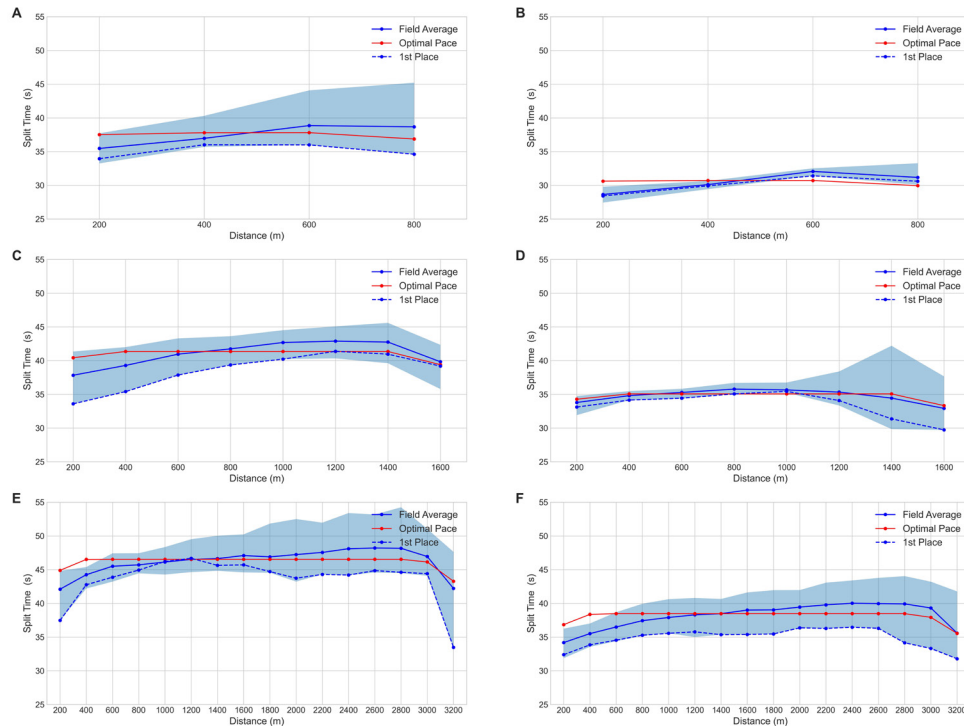


Figure 3: Comparison of high school athlete performance to optimal pacing approach. Graph of the official split times for the (A) girls' 800 m, (B) boys' 800 m, (C) girls' 1600 m, (D) boys' 1600 m, (E) girls' 3200 m, and (F) boys' 3200 m indoor distance events. The field average (solid blue lines) and 1st place finisher (dotted blue lines) split times are shown in comparison to the optimal pacing approach (solid red line). The range bounded by the minimum and maximum recorded split times is illustrated within the shaded region. Computer simulation of the energy depletion model was used to generate the optimal split times for each of the girls' and boys' distance events. Physiological model parameters were identified using a steady-state analysis and fine-tuned using the field average finishing time.

	Boys' 800 m	Girls' 800 m	Boys' 1600 m	Girls' 1600 m	Boys' 3200 m	Girls' 3200 m
Start phase distance	102 m	97 m	184 m	170 m	307 m	259 m
Middle phase distance	586 m	593 m	1198 m	1218 m	2484 m	2565 m
Finishing phase distance	112 m	110 m	218 m	212 m	409 m	376 m

Table 4: Distance covered during the start, middle, and finishing phases of high school events.

that starting faster at the beginning of a race could jumpstart the aerobic metabolism, helping achieve maximal oxygen uptake ($VO_2\max$) earlier while reducing the anaerobic energy consumed during the beginning phase of the race (10, 14). This initial distance can be adjusted by altering the accumulated oxygen deficit threshold, γ_s , if runners require more time to establish position. However, changing this transition point will also affect the maximal attainable velocity during the middle phase of the race. This final sprinting phase begins at the γ_f anaerobic threshold and rapidly brings the runner to full muscle energy depletion. Theoretically, the anaerobic energy left in the muscles at the end of the race should reach zero.

The simulated and actual split times deviated greatly during the elite 800 m events, in which neither the men nor women competitors set Olympic record performances. In both cases, the male and female winners began with a faster start and dropped to a slower than optimal pace during the middle phase of the race. The simulated split times were a closer match to the actual race split times for both the women's and men's 1500 m events, during which both set Olympic records. The 1500 m event gold medalists exhibited a fast start through

the opening 200 m before settling into a constant pace. The main time discrepancy occurred when both runners exhibited an earlier final kick than what the optimal pacing approach suggested. If the competitors' anaerobic energy levels were not approaching zero during this final phase, then a faster finish would be achievable.

At the high school level, many top-seeded runners use a front running strategy by running at their maximal ability and trying to win by setting new PRs rather than opting for a more patient approach (3). Our hypothesis predicted that slower seeded runners would try to match the fast pace set by the top seeded runners, thereby causing them to deviate from the optimal pacing profile they could follow based on their own individual capabilities. The field average split times increased during the middle phase of the race in the girls' and boys' 800 m, the girls' 1600 m, and the girls' and boys' 3200 m races. Only the boys' 1600 m field average exhibited the optimal characteristics of a fast start, even pacing middle phase, and a fast finish. Likewise, only the first-place runner of the 800 m girls' race exhibited even pacing characteristics over the middle phase.

Analysis of split times from indoor track events indicated that the average time performance of the competition field was more likely to not follow the optimal pacing approach, supporting our hypothesis. In a competitive environment, runners tend to start quickly and attempt to hold a fast pace to achieve a high finish placement. This approach often led to early onset of fatigue and eventually resulted in a gradual reduction in pace. In 5 of 6 races we analyzed, the average pace of the field was significantly ahead of the optimal split times in the early stage of the race. Any time gained here was eventually lost as the field slowed during the middle phase of the race. In the one other race, the boys' 1600 m competitors all held similar performance capabilities with each other, resulting in a close-packed field running near the theoretical optimal pace profile.

We estimated physiological parameters using a steady-state analysis and compared to those provided in published literature (9, 11, 12, 13). There is no unique set of parameters necessary to achieve a given finishing time performance. Future work involving the minimization of least squares between the simulated and actual split times could be investigated to further tune these parameters. Alternatively, individual athlete characteristics can be identified using a combination of short 80 m sprints and longer 1600 m time trials (12). Accurately identifying these parameters would allow the model simulation to provide a set of achievable split times for a particular athlete's capabilities. Moreover, a coach may structure threshold workouts at 85–95% of the optimal pace profile to help train for competition.

Simulating variations in the physiological parameters also provides coaches and athletes with insight into how a change in one parameter can affect performance (12). For example, improving oxygen uptake and running economy both allow a runner to maintain a higher mean velocity (12). Increasing the maximal propulsive force increases the peak velocity achieved during the beginning and end of the race. A larger initial stored anaerobic energy allows the exertion of a bigger propulsive force and a higher velocity at the end of the race. Strength training can also improve running economy and the maximal propulsive force, without affecting oxygen uptake (15). Runners can use interval training and fartlek workouts – where the pace alternates between a fast and slow tempo – to improve the ability to transition between the three race phases (16).

Human psychology, prior experience, and an athlete's training all play a role in determining how a runner may change their effort during a race (7). Based on this information, the mind can estimate how much energy the body has remaining versus the distance left to cover. However, surrounding competitors who create a surge in pace early may draw others away from the time optimal approach. In fact, runners who surge too early risk burning out in the final 100 m (10). Having knowledge and understanding of the optimal pacing approach benefits runners who stay patient and trust their abilities.

The main contribution of this study was to help middle and high school track athletes gain a better understanding of how to manage their limited energy resources and achieve a PR. The analysis presented in this paper focused on running for time, as opposed to aiming to win using strategic racing methods. Simulation of the energy depletion model and comparison to recorded split times provides coaches with a

useful tool for analyzing an individual athlete's capabilities. With this knowledge, coaches may then address the athlete's weaknesses through specifically tailored workout routines, as well as provide guidance on how to transition their pace across the different phases of the race.

MATERIALS AND METHODS

Data Sources

Olympic and world record times can be found on the World Athletics website (1). Listed on the website's results are the official 100 m split times for the men's and women's 800m and 1500m events—along with all other track events—from the Tokyo 2020 Olympics Games, where Jakob Ingebrigtsen and Faith Kipyegon set Olympic records of 3:28.32 and 3:53.11 for the men's and women's 1500m events, respectively. Split times from high school track meets are often not available, with only finishing times usually reported from a fully automated timing (FAT) system. Fortunately, available data allowed us to analyze official split times for every 200 m lap at an indoor track meet consisting of 81 high school athletes who met specific qualifying standards for the 800m, 1600m, and 3200m events (17). Rather than comparing the optimal approach against each individual, we chose to compare it against the average pace of the field of runners competing in the event. The spread of split times was found by capturing the minimum and maximum split time from all the competitors over each interval. One or more runners may have indeed closely followed the optimal pacing approach. However, since finishing placement at this track meet was of high importance, the field average and split time spread indicated whether several of the competitors started at an unsustainable pace. The percent difference from the optimal pace profile was calculated at each split interval by taking the absolute value of the model minus the actual split times and dividing by the model split time.

Simulation Model

A mathematical formulation of the distance pacing problem is defined as minimizing the time T to run a race distance D with velocity profile $v(t)$, and is written as (9)

$$D = \int_0^T v(t)dt \quad (\text{Equation 1})$$

The velocity $v(t)$, with initial starting condition $v(0) = 0$, satisfies Newton's second law of motion (acceleration = force/mass) with the inclusion of a resistive force per unit mass, $v(t)/\tau$, such that

$$dv(t)/dt + v(t)/\tau = f(t) \quad (\text{Equation 2})$$

The physiological attribute τ corresponds to the athlete's running economy—the energy cost associated with running at a given velocity. The time-dependent profile, $f(t)$, is the runner's propulsive force per unit mass such that $0 \leq f(t) \leq f_{max}$, with f_{max} being the maximum attainable force. The propulsive force also affects the amount of anaerobic energy per unit mass contained in the muscles, $e(t)$, with initial energy $e(0) = e_0$. The anaerobic energy in the working muscles, supplied aerobically through oxygen consumption, $\sigma(t)$, is given by

$$de(t)/dt = \sigma(t) - f(t)v(t) \quad (\text{Equation 3})$$

The rate at which oxygen is supplied to the muscles through breathing and blood circulation, $\sigma(t)$, has been captured using a hydraulic container model of the anaerobic metabolism process (12, 13, 18). Here, the oxygen uptake rate depends on the level of anaerobic energy in the muscles during three phases of the race (start, middle, and finish):

$$\sigma(t) = \begin{cases} \sigma_s + (\sigma_m - \sigma_s)(1 - e(t)/e_0)/\gamma_s & \text{if } 1 - e(t)/e_0 \leq \gamma_s \\ \sigma_m & \text{if } 1 - e(t)/e_0 > \gamma_s \text{ and } e(t)/e_0 \geq \gamma_f \\ \sigma_f + (\sigma_m - \sigma_f)(e(t)/e_0)/\gamma_f & \text{if } e(t)/e_0 < \gamma_f \end{cases}$$

(Equation 4)

The constant σ_s is the resting oxygen uptake rate at the start of the race, σ_f is the oxygen uptake rate at the finish of the race, and σ_m is the maximum oxygen uptake rate experienced during the race. An individual's oxygen uptake rate is limited by their body's ability to transport and utilize oxygen in the muscles and depends on their current exertion level (15). In the early phase of the race, the oxygen rate transitions from its resting value to its maximum value as soon as the accumulated oxygen deficit reaches a critical value γ_s . In the final phase of the race, the maximal oxygen rate drops due to limitations when the anaerobic energy supply reaches a critical fraction of its initial stored energy, γ_f (13).

The energy depletion model contains differential equations governing Newton's second law (Equation 2) and the change in anaerobic muscle energy (Equation 3), accompanied by a piecewise function depicting the oxygen uptake process (Equation 4). To apply this model to our analysis, we created a computer simulation using Python. This code allowed the adjustment of model parameters until the simulation matched the official finishing time. The differential equations were simulated by looping through a range of time increments, with each iteration increasing by $dt = 0.01$ seconds. This allowed the calculation of the velocity derivative using the discrete approximation,

$$\begin{aligned} vdot &= f[t] - v[t]/\tau \\ v[t + 1] &= v[t] + vdot * dt \end{aligned}$$

with the energy derivative term calculated using the same technique. The simulation loop ended when either the distance traveled reached the event distance or when the anaerobic energy reached zero. Additional code was written to identify the split times within the simulation, along with the distances traveled within the initial and final phases of the race. Graphical figures were generated using the matplotlib library (19).

The optimal propulsive force input profile begins with a fast start at maximal effort for up to two seconds, transitioning smoothly to a constant effort through the middle phase of the race. The propulsive force then smoothly transitions back towards maximal effort over the final sprinting phase (13). We decided to model the runner's theoretical optimal effort into three separate components. The starting phase of the race begins at maximal effort and decreases to a constant propulsive force through the sinusoidal function,

$$f_{start}(t) = 0.5f_{max} * (1 + r - (1 - r)\cos(\pi(1 - e(t)/e_0)/\gamma_s))$$

(Equation 5)

Here, we chose the runner to reach a sustainable fraction, $0 < r \leq 1$, of the maximal effort level, $f_{middle}(t) = r * f_{max}$, during the middle phase of the race (i.e., race pace effort). This transition point occurs as soon as the stored anaerobic energy reaches

the γ_s anaerobic energy depletion threshold. When the ratio of anaerobic energy to initial stored muscle energy reaches the γ_f threshold near the end of the race, the runner then begins to increase towards maximal effort through the sinusoidal function,

$$f_{finish}(t) = 0.5f_{max} * (1 + r - (1 - r)\cos(\pi e(t)/e_0\gamma_f))$$

(Equation 6)

Physiological Parameters

The Respiratory Exchange Ratio estimates 1 L of oxygen uptake to produce 21.1 kJ of energy expenditure (20). An individual's $VO_2\max$ is often reported in ml/kg/min, and after a unit conversion to m^2/s^3 , the oxygen consumption (per kg mass) becomes $\sigma_m = VO_2\max/2.84$. Elite men and women exhibit $VO_2\max$ levels in the range of 65–80 ml/kg/min, while conditioned high school athletes can be in the range of 50–65 ml/kg/min (21, 22). For example, our estimate of $VO_2\max$ in the men's 1500m Olympic record performance was 74 ml/kg/min, or $\sigma_m = 26 m^2/s^3$. The initial oxygen uptake rate was set at $\sigma_s = 6 m^2/s^3$, while the oxygen uptake rate over the final stage of the race was estimated to drop by 10% (i.e., $\sigma_f = 0.9\sigma_m$) (11).

From the two differential equations in the energy depletion model, the steady-state equilibrium was obtained by setting the velocity and energy derivative terms to zero. The resulting steady-state velocity, v_{ss} , and propulsive force, f_{ss} , terms become $v_{ss} = \sqrt{\sigma_m\tau}$ and $f_{ss} = \sqrt{\sigma_m/\tau}$, respectively.

This equilibrium condition exists when the aerobic energy supplied by the oxygen uptake balances the anaerobic energy expended by the muscles, while the resistance experienced due to the running economy balances the applied propulsive force. An elite 1500m competitor can run a 10 km threshold training run under 34 minutes, leading to a sustained average pace of 4.92 m/s (11 mph) (22). Using the steady-state velocity equation, the resistance term related to running economy becomes $\tau = v_{ss}^2/\sigma_m = 0.929$ s. The corresponding steady-state propulsive force (per kg mass) to maintain this pace is

$$f_{ss} = \sqrt{\sigma_m/\tau} = 5.3 m/s^2$$

Based on parameters used in prior literature for the men's 1500m world record performance ($\sigma_m = 22$, $\tau = 0.932$, $f_{max} = 8$), the steady state force was

$$\sqrt{\sigma_m/\tau}/f_{max} = \sqrt{22/0.932}/8$$

= 61% of the maximum applied force (11).

We found that a value of $f_{max} = f_{ss}/0.62 = 8.55 m/s^2$, combined with an initial stored energy value of $e_0 = 6519 m^2/s^2$ allowed the model simulation to match the 1500m Olympic record time with near zero energy remaining at the finish. The oxygen uptake transition thresholds, $\gamma_s = \gamma_f = 0.15$, were used for all events. A similar set of steady-state calculations was performed to create parameter estimates for high school athletes based on $VO_2\max = 50$ –65 ml/kg/min and 10 km threshold times of 37–42 minutes.

The steady-state analysis provided a starting point for determining the four physiological parameters for each event. Fine tuning these parameters to simulate an identical finishing time to the runner's actual performance began by fixing the estimate of the maximum oxygen uptake rate, σ_m , then adjusting the running economy, τ , and the maximum propulsive force, f_{max} , parameters. The initial stored anaerobic

energy value estimate, e_o , was set 20% higher to ensure the simulation did not run out of energy prior to finishing the event distance. If the simulated finish time was faster than the actual time, then the running economy value was first decreased slightly, followed by a slight decrease in the propulsive force. The aim of adjusting these two parameters was to obtain a simulated finish time approximately 0.2–0.5 seconds slower than the actual finish time. Next, the initial stored energy parameter, e_o , was decreased until the remaining energy at the finish was as near to zero as possible. This adjustment maximized the time spent in the final phase of the race where the propulsive force profile increased to generate the final kick. If the simulated finish time was then faster (slower) than the actual time, the running economy and propulsive force values were decreased (increased) slightly again. This process was iterated until the simulated and actual finish times were identical and the stored energy depleted to zero.

ACKNOWLEDGEMENTS

We would like to thank the high school coaches at Sacred Heart-Griffin High School for sharing their knowledge and expertise during practices, and to thank the parents of Kaitlyn Lee for their endless support and caring advice. We would also like to thank the JEI editor and reviewers for their valuable feedback on improving the content of this paper.

Received: June 3, 2022

Accepted: August 13, 2022

Published: November 28, 2022

REFERENCES

1. "Olympic Games – Results." *World Athletics*. www.worldathletics.org/results/olympic-games. Accessed 10 Sept. 2022.
2. Mantzios, Konstantinos, et al. "Effects of weather parameters on endurance running performance: discipline-specific analysis of 1258 races." *Medicine and Science in Sports and Exercise*, vol. 54, no. 1, Jan. 2022, pp. 153-161, doi:10.1249/MSS.0000000000002769.
3. Makubuya, Timothy and Micky Kaufman. "Racing strategies and tactics – for outdoor middle distance events." *Techniques Magazine*, vol. 12, no. 1, Sept. 2018, pp. 18-26, U.S. Track & Field and Cross Country Coaches Association.
4. Schickhofer, Lukas and Henry Hanson. "Aerodynamic effects and performance improvements of running in drafting formations." *Journal of Biomechanics*, vol. 122, Jun. 2021, doi:10.1016/j.jbiomech.2021.110457.
5. Aragón, Sonia, et al. "Tactical behaviour of winning athletes in major championship 1500-m and 5000-m track finals." *European Journal of Sport Science*, vol. 16, no. 3, 2016, pp. 279-286, doi:10.1080/17461391.2015.1009494.
6. Renfree, Andrew, et al. "Tactical considerations in the middle-distance running events at the 2012 Olympic Games: a case study." *International Journal of Sports Physiology and Performance*, vol. 9, no. 2, Mar. 2014, pp. 362-364, doi:10.1123/ijsp.2013-0020.
7. Tucker, Ross, et al. "An analysis of pacing strategies during men's world-record performances in track athletics." *International Journal of Sports Physiology & Performance*, vol. 1, no. 3, Sept. 2006, doi:10.1123/ijsp.1.3.233.
8. Aftalion, Amandine. "How to run 100 meters." *SIAM Journal on Applied Mathematics*, vol. 77, no. 4, Jun. 2017, pp. 1320-1334, doi:10.48550/arXiv.1606.09497.
9. Keller, Joseph B. "Optimal velocity in a race." *The American Mathematical Monthly* vol. 81, no. 5, May 1974, pp. 474-480, doi:10.2307/2318584.
10. Hanon, Christine, et al. "Pacing strategy and VO2 kinetics during a 1500-m race." *International Journal of Sports Medicine*, vol. 29, no. 3, Mar. 2008, pp. 206-211, doi:10.1055/s-2007-965109.
11. Aftalion, Amandine and Emmanuel Trélat. "Pace and motor control optimization for a runner." *Journal of Mathematical Biology*, vol. 83, no. 1, May 2021, pp. 1-21, doi:10.48550/arXiv.2005.09882.
12. Aftalion, Amandine, et al. "How to identify the physiological parameters and run the optimal race." *Mathematics In Action*, vol. 7, no. 1, 2016, pp. 1-10, doi:10.5802/msia.9.
13. Aftalion, Amandine and J. Frédéric Bonnans. "Optimization of running strategies based on anaerobic energy and variations of velocity." *SIAM Journal on Applied Mathematics*, vol. 74, no. 5, 2014, pp. 1615-1636, doi:10.1137/130932697.
14. Jones, A. M., et al. "Influence of pacing strategy on O2 uptake and exercise tolerance." *Scandinavian Journal of Medicine & Science in Sports*, vol. 18, no. 5, Oct. 2008, pp. 615-626, doi:10.1111/j.1600-0838.2007.00725.x.
15. Blagrove, Richard C., et al. "Effects of strength training on the physiological determinants of middle-and long-distance running performance: a systematic review." *Sports Medicine*, vol. 48, no. 5, May 2018, pp. 1117-1149, doi: 10.1007/s40279-017-0835-7.
16. Thiel, Christian, et al. "Pacing in Olympic track races: competitive tactics versus best performance strategy." *Journal of Sports Sciences*, vol. 30, no. 11, 2012, pp. 1107-1115, doi:10.1080/02640414.2012.701759.
17. "Illinois Top Times (Class 1A)." *AdkinsTrak*. results.adkinstrak.com/meets/12848. Accessed 10 Sept. 2022.
18. Reardon, James. "Optimal pacing for running 400-and 800-m track races." *American Journal of Physics*, vol. 81, no. 6, 2013, pp. 428-435, doi:10.1119/1.4803068.
19. "Matplotlib: visualization with Python." *Matplotlib*. matplotlib.org. Accessed 10 Sept. 2022.
20. Scott, Christopher. "Misconceptions about aerobic and anaerobic energy expenditure." *Journal of the International Society of Sports Nutrition*, vol. 2, no. 2, 2005, pp. 1-6, doi:10.1186/1550-2783-2-2-32.
21. Magness, Steve. *The Science of Running: How to Find Your Limit and Train to Maximize Your Performance*, Origin Press, 2014.
22. Vigil, Joe I. *Road to the Top: A Systematic Approach to Distance Training that Produced One of America's Greatest Running Programs*, edited by Doug Lindsey, Creative Designs, Inc., 1995.

Copyright: © 2022 Lee and Lee. All JEI articles are distributed under the attribution non-commercial, no derivative license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>). This means that anyone is free to share, copy and distribute an unaltered article for non-commercial purposes provided the original author and source is credited.