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Article

Comparing measurements of Sun-Earth distance: Shadow method and two pinhole method variations

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SUMMARY

This study compares three methods, one using a newly derived formula (the shadow method) and two conventionally used (the plate pinhole method and tube pinhole method), regarding their accuracy in calculating the distance between the Earth and the Sun. In doing so, the new formula will add to the existing knowledge surrounding viable methods to calculate the distance between the Earth and Sun, focusing primarily on providing insight on methods to make astronomical observation more accessible to the casual observer. The tube and plate pinhole methods were chosen due to their promotion by reputable organizations and use in schools, to best compare to and evaluate the utility of the shadow method in everyday use. Our hypothesis was that the shadow method would have the greatest mean accuracy, followed by the tube pinhole method, and finally the plate pinhole method. Measurements of the shadow and pinhole methods were each carried out multiple times over a day, and the values were inputted into each method's formula to calculate the estimated Sun-Earth distance and error. Our results validate the hypothesis; however, further investigation would be helpful in determining effective mitigation of each method's limitations and the effectiveness of each method in determining the distance of other light-emitting objects distant from the Earth.

INTRODUCTION

Astronomical observation often requires complex tools and techniques, which can make observations difficult for ordinary people to complete. In fact, since 1961, astronomers have used radar to measure interplanetary distances, and prior to 1961, geometric methods, such as Parallax, were used to approximate these distances (1). However, the substantial technology and preparations necessary to use either of these techniques illustrate the need for simpler methods to calculate astronomical distances in order to make astronomical observation more accessible for the general population (1, 2). For example, the 1692 calculation of the Sun-Earth distance using Parallax required simultaneous observations over 7,000 km apart (1, 2).

Two commonly used methods to determine interplanetary distances are the plate and tube pinhole methods, which use measurements of a projection of the Sun through either a plate or tube pinhole projector to calculate the Sun-Earth distance (3, 4). The use of the pinhole method is supported by NASA and in one project, is labeled for ages 10 and up, illustrating the utility of the pinhole method among the general public (5). However, these methods are tedious to implement. The plate pinhole projector requires the prior manual construction of the projector using heavy cardboard and aluminum foil and is most accurate on a clear day (3). This method is also recommended to be conducted with two people (5). The tube pinhole projector also requires prior construction, as well as a cardboard tube at least 30 in long, mm-spaced graph paper, and aluminum foil – materials that may not be immediately available in the standard home or school. Furthermore, these pinhole methods have various drawbacks that restrict their accuracy. Firstly, the projected image of the Sun can be too dim to effectively measure (3). Additionally, both the plate pinhole projector and the paper onto which the Sun's light projects is expected to be held up by the experimenters, increasing the likelihood of erroneous measurement due to subtle variations of the projector and paper's positions (5). Finally, due to the diameter of the pinhole being neglected in calculations, the pinhole is viewed as the convergence point of the Sun's rays, whereas the convergence point is actually above the pinhole, illustrating a mathematical complication neglected in both pinhole methods (**Figure 1**) (5). Therefore, the conventional pinhole methods used by the public for determining the Sun-Earth distance have various difficulties, illustrating the need for a simpler and more accurate method to allow the casual observer to perform astronomical calculations without the difficulty and inaccuracy present in the pinhole methods.

Using a newly derived formula using object and shadow length measurements to calculate the Sun-Earth distance, we seek to evaluate the effectiveness of using this formula to achieve our goal. More specifically, we seek to evaluate the hypothesis of whether the newly derived formula (the shadow method) can calculate the distance between the Sun and the Earth with greater accuracy than either pinhole method, along with the prediction that the tube pinhole projector will be more accurate than the plate pinhole projector. Not only does the shadow method require fewer materials and no construction, but it also lacks any drawbacks outlined above in the pinhole

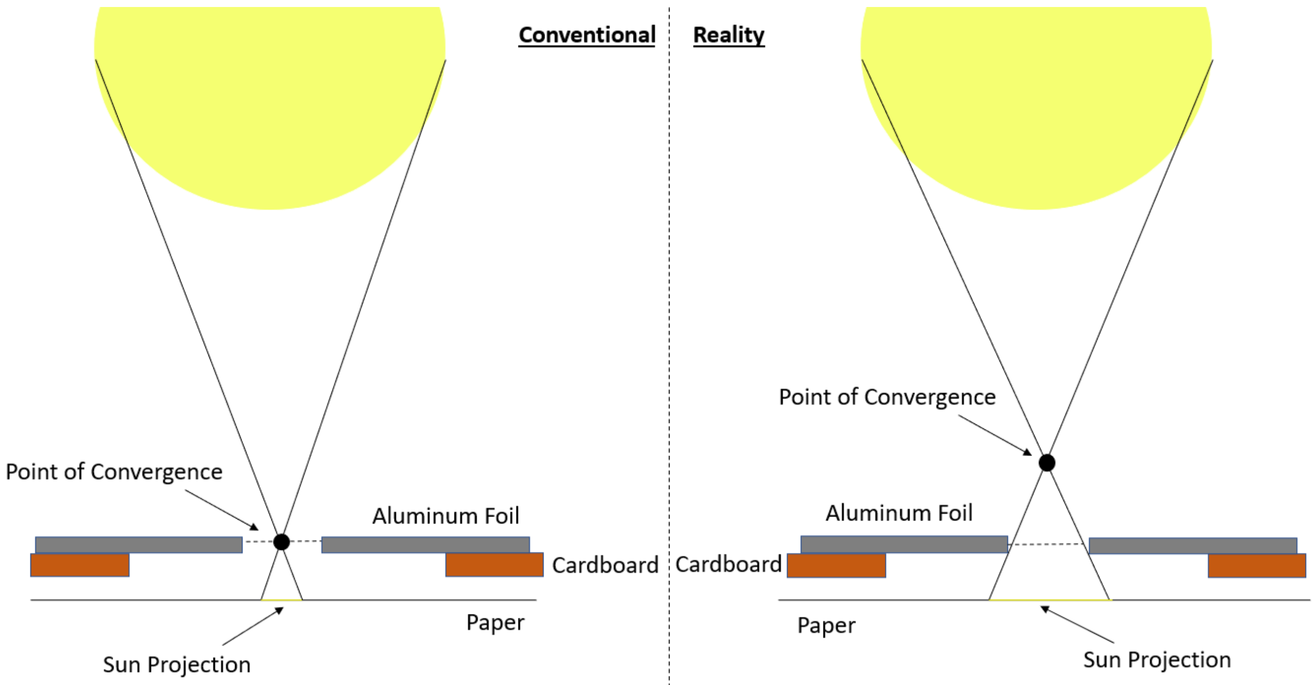

Figure 1. Conventional and real diagrams of the pinhole method. The line segments in the diagram are used to illustrate how the conventional and real position of the point of convergence of light rays and the length of the projection of the Sun are different, illustrating a neglected mathematical complication in the pinhole method. Not to scale.

methods: none of its measurements are dim (due to the lack of projected light), there are no variations in positions as all materials are propped up on solid surfaces, and no significant mathematical errors are present. While the tube projector carries notable disadvantages, it is still predicted to be superior to the plate projector due to the mitigation of various causes of errors, such as dim projections and positional variations. Data from experimentation validates both parts of the hypothesis, identifying the shadow method as the most accurate method, followed by the tube projector, and finally the plate projector. Further research has significant potential to reduce the margin of error and investigate all methods' applicability to other light-emitting bodies, such as the Moon.

RESULTS

In order to compare the abilities of the shadow and pinhole methods to calculate the distance between the Sun and Earth, it is necessary to collect empirical data to evaluate and compare each method's practical accuracy.

For the shadow method, after deriving the new formula, a card was placed on raised surfaces and three measurements were taken of each card and its shadow. These values were

Table 1. Measurements, errors, and Sun-Earth distance calculations for all samples using Shadow method.

Sample #	Time	Length of Object (cm)	Length of Shadow (cm)	Distance from Shadow Corner to Corresponding Object Corner (cm)	Calculated Sun-Earth Distance using Formula (cm)	Error (%)
1	8:46 AM	13.2	12	125	1.45×10^{13}	4.62
2	8:50 AM	13.2	12.1	123	1.56×10^{13}	2.39
3	9:16 AM	13.2	12.3	104.3	1.61×10^{13}	6.11
4	9:44 AM	13.2	12.4	91.7	1.60×10^{13}	4.96
5	11:15 AM	13.2	12.6	67	1.56×10^{13}	2.25
6	12:00 PM	13.2	12.7	62.5	1.74×10^{13}	14.46
7	2:38 PM	13.2	12.7	47.5	1.32×10^{13}	13.01
8	4:20 PM	13.2	12.8	57.7	2.01×10^{13}	32.08
9	5:02 PM	13.2	12.6	65.8	1.53×10^{13}	0.42
10	6:03 PM	13.2	12.4	87	1.51×10^{13}	0.41
Mean		13.2	12.46	83.13	1.59×10^{13}	8.07

Table 2. Individual and mean ideal plate pinhole method measurements and errors.

Sample #	Time	Diameter of Sun Image (cm)	Distance from Image to Pinhole (cm)	Calculated Sun-Earth Distance using Pinhole Formula (cm)	Error (%)
1	8:05 AM	0.3	32.1	1.49×10^{13}	0.44
2	9:42 AM	0.1	12.5	1.74×10^{13}	16.31
3	11:00 AM	0.1	17.5	2.44×10^{13}	62.83
4	12:03 PM	0.2	15.9	1.11×10^{13}	26.03
5	1:20 PM	0.2	28.5	1.98×10^{13}	32.59
6	2:47 PM	0.3	40.5	1.88×10^{13}	25.61
7	5:35 PM	0.3	20	9.28×10^{12}	37.97
8	7:42 PM	0.2	25.5	1.77×10^{13}	18.64
Mean		0.2125	24.625	1.67×10^{13}	27.55

then substituted, along with the diameter of the Sun, into the formula in order to evaluate its accuracy.

For the pinhole methods, two cardboard pinhole projectors (a plate projector and a tube projector) were constructed and appropriately angled, along with a paper surface, so as to create a projection of the Sun. In each sample, two measurements were taken of the projection and substituted, along with the Sun's diameter, into the method's formula to evaluate its accuracy. Eight sample measurements were collected for each method.

For the shadow method, calculated Sun-Earth distance was relatively accurate with a mean error of 8.07% (**Table 1**). While Sample 8 had the greatest error of 32.08%, the substantially smaller mean error suggests that such results are a rarity, and that the formula generally is much more accurate than the outlier suggests (**Table 1**).

For the plate projector, the calculated Sun-Earth distance was substantially less accurate with a mean error of 27.55%, which is over 3 times less accurate than the shadow method (**Tables 1, 2**). While Sample 1 of the plate projector had an unusually low error of 0.44%, the substantially greater mean error suggests that such results are a rarity, and that the pinhole method's formula is generally much more inaccurate than the outlier suggests (**Table 2**).

For the tube projector, the calculated Sun-Earth distance was relatively accurate with a mean error of 10.92%, which is almost 3 times more accurate than the plate projector but slightly less accurate than the shadow method (**Tables 1-3**). There were no notable outliers, with all the error percentages being either 5.56% or 14.14%. Overall, based on the mean error, the shadow method was the most accurate method to determine the distance between the Sun and Earth, followed closely by the tube projector and farther behind, the plate projector, validating the hypothesis.

DISCUSSION

The measurements collected from the shadow and pinhole methods samples reveal the overall superior accuracy of the shadow method and the greater accuracy of the tube pinhole method over the plate pinhole method, validating our hypothesis. In other words, the mean error of the shadow method's samples was less than that of the tube pinhole method, which was less than that of the plate pinhole method. Therefore, both pinhole methods are more inaccurate than the shadow method in determining the Sun-Earth distance. Although the 8.07% accuracy of the shadow method is generally insufficient for the standards of scientific rigor (radar allows scientists to determine interplanetary distances with uncertainties of only a few thousandths of a percent), a mean accuracy of under 10% is more than sufficient for casual scientists to understand the sheer scale of interplanetary distances and perform casual observation to a reasonable degree of accuracy (6). More importantly, the shadow method is both easier to use and more accurate than the conventional

Table 3. Individual and mean ideal tube pinhole method measurements and errors.

Sample #	Time	Diameter of Sun Image (cm)	Distance from Image to Pinhole (cm)	Calculated Sun-Earth Distance using Pinhole Formula (cm)	Error (%)
1	8:52 AM	1.1	101.5	1.28×10^{13}	14.14
2	10:11 AM	1.0	101.5	1.41×10^{13}	5.56
3	11:15 AM	1.1	101.5	1.28×10^{13}	14.14
4	12:20 PM	1.1	101.5	1.28×10^{13}	14.14
5	1:45 PM	1.1	101.5	1.28×10^{13}	14.14
6	2:58 PM	1.0	101.5	1.41×10^{13}	5.56
7	4:11 PM	1.1	101.5	1.28×10^{13}	14.14
8	5:09 PM	1.0	101.5	1.41×10^{13}	5.56
Mean		1.0625	101.5	1.33×10^{13}	10.92

Table 4. Individual and mean ideal shadow lengths and errors.

Sample #	1	2	3	4	5	6	7	8	9	10	Mean
Ideal Length of Shadow (cm)	12.055	12.074	12.245	12.360	12.587	12.628	12.765	12.672	12.598	12.403	
Shadow Length Error (cm)	0.055	0.026	0.055	0.040	0.013	0.072	0.065	0.128	0.002	0.003	0.0459

pinhole methods used by the general public for calculating the Sun-Earth distance, providing a better way for casual observers to perform astronomical observation. Additionally, all methods used have the potential for much greater accuracy when applied with greater scientific rigor, potentially making them viable for serious scientific observation (**Tables 4, 5**). All measurements taken in each sample were done using a standard wooden ruler, which is only accurate to within 1mm. For the shadow, plate pinhole, and tube pinhole methods, the mean shadow/projection measurement errors were 0.0459, 0.052, and 0.118 cm respectively, which are either well under 1mm or close to 1mm (**Tables 4, 5**). Furthermore, the shadow and object measurement uncertainty for the shadow method is 27.03% (0.2 / (mean object length – mean shadow length)), and projection measurement uncertainty (0.2 / mean projection diameter) for the plate projector is 94.12% and 18.82% for the tube projector (**Tables 1-3**). Since the measurement errors are close to or under 1mm and the measurement uncertainties are significant, it is plausible that each method's error in the Sun-Earth distance calculations was primarily due to the accuracy limitation inherent in the ruler, and that given more precise measurement devices, significant error present in experimentation would be eliminated. The plate pinhole method especially would stand to benefit from an improvement in precision, considering its astronomically high projection measurement uncertainty of 94.12%. While it is clear that a significant potential cause of error and therefore a significant step to improving the accuracy of these methods lie in improving measurement precision, it is equally important to understand that 1mm precision is often the most precise a student or casual observer can afford. Therefore, the evaluation of the most effective method for general use should be done through data collected with low-precision devices, not from the high-caliber tools available to professionals.

Another potential cause of experimental error in the shadow method is the inaccurate measurements of the corners of the objects to the corresponding corners of their shadows. In experimentation, the corner measurements were taken after the shadow length measurements. As the distance of the shadow from the object changes depending on the Sun's position in the sky, it is plausible that the distance of the shadow from the object changed slightly in the time between the shadow length measurement and the corner measurement, causing a slightly inaccurate corner measurement to be recorded. However, any potential change in the corner measurement would have likely been insignificant due to the passing of mere seconds between the two measurements, making it a largely insignificant factor in the error, especially in comparison to the low precision of the ruler used in the experiment.

Regarding the plate pinhole method, there are numerous probable causes of error, which likely explain the method's relative inaccuracy in comparison to the shadow method. As

Table 5. Individual and mean ideal pinhole projection diameters and errors.

Pinhole Projector Type	Value	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7	Sample 8	Mean
Plate	Ideal Projection Diameter (cm)	0.294	0.114	0.160	0.146	0.261	0.371	0.183	0.233	
Plate	Projection Diameter Error (cm)	0.006	0.014	0.060	0.054	0.061	0.071	0.117	0.033	0.052
Tube	Ideal Projection Diameter (cm)	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.945	
Tube	Projection Diameter Error (cm)	0.155	0.055	0.155	0.155	0.155	0.055	0.155	0.055	0.118

previously stated, the projected image of the Sun can be too dim to effectively measure (3). Additionally, both the pinhole projector and the paper onto which the Sun's light projects is expected to be (and was) held up by the experimenters, which increases the likelihood of erroneous measurement due to subtle variations of the projector and paper's positions (5). This could be corrected with specialized supports to hold up the projectors; however, it is unlikely that such tools will be commonly used by the public in casual observation. Finally, due to the diameter of the pinhole being neglected in calculations, the pinhole is viewed as the convergence point of the Sun's rays, whereas the convergence point is above the pinhole, illustrating a neglected mathematical complication in the pinhole method that should be addressed (**Figure 1**) (5).

The tube pinhole method is much more accurate than the plate pinhole method, possibly because it suffers from fewer sources of error. The problem of a dim projection is largely resolved by using a small viewing hole that prevents substantial excess light that could reduce contrast. The projector is also stabilized by a solid surface (in this case, a chair), meaning that there is generally no need to hold anything and there is less shaking, ensuring less erroneous measurements. While it is true that a slightly larger pinhole diameter of 1mm was used for this projector, larger than the recommended single prick of a pin, and that larger pinholes are known to lead to more out of focus images and less accurate results, the pinhole enlargement was very small in our study (approximately 0.25 mm increase in diameter) and the image was still clear to measure, making this an insignificant source of error (4). In fact, a larger pinhole creates a larger projection of the Sun, which, due to a greater diameter being measured, would reduce measurement uncertainty, potentially improving the accuracy of the method (3). However, other sources of error remain. In Samples 4 and 5, the sharp angle of the projector required the projector to be held during those trials and the slight shaking could cause increased inaccuracy in those measurements, which could be corrected with special supports that the general public may find too much of a nuisance to use. Finally, as stated above with the plate projector, the neglection of the diameter of the pinhole in calculations would yield an inaccurate result. Therefore, while the tube pinhole projector mitigates many causes of error present in the plate pinhole projector, it still has some causes of error that inhibit its accuracy.

Further investigation is necessary to verify these techniques in order to mitigate the errors present in all methods as well as the methods' applicability to other distant bodies in the solar system. One potential subject of study is the Moon, which can be bright enough to cast visible shadows, making it a valid focal point of experimentation.

The final topic of interest is the shadow formula (refer to **Figure 2** for symbols). The variable $\overline{AE}$ is technically an approximation of the true distance of the Sun from the Earth since it measures the distance from the Earth to the Sun's

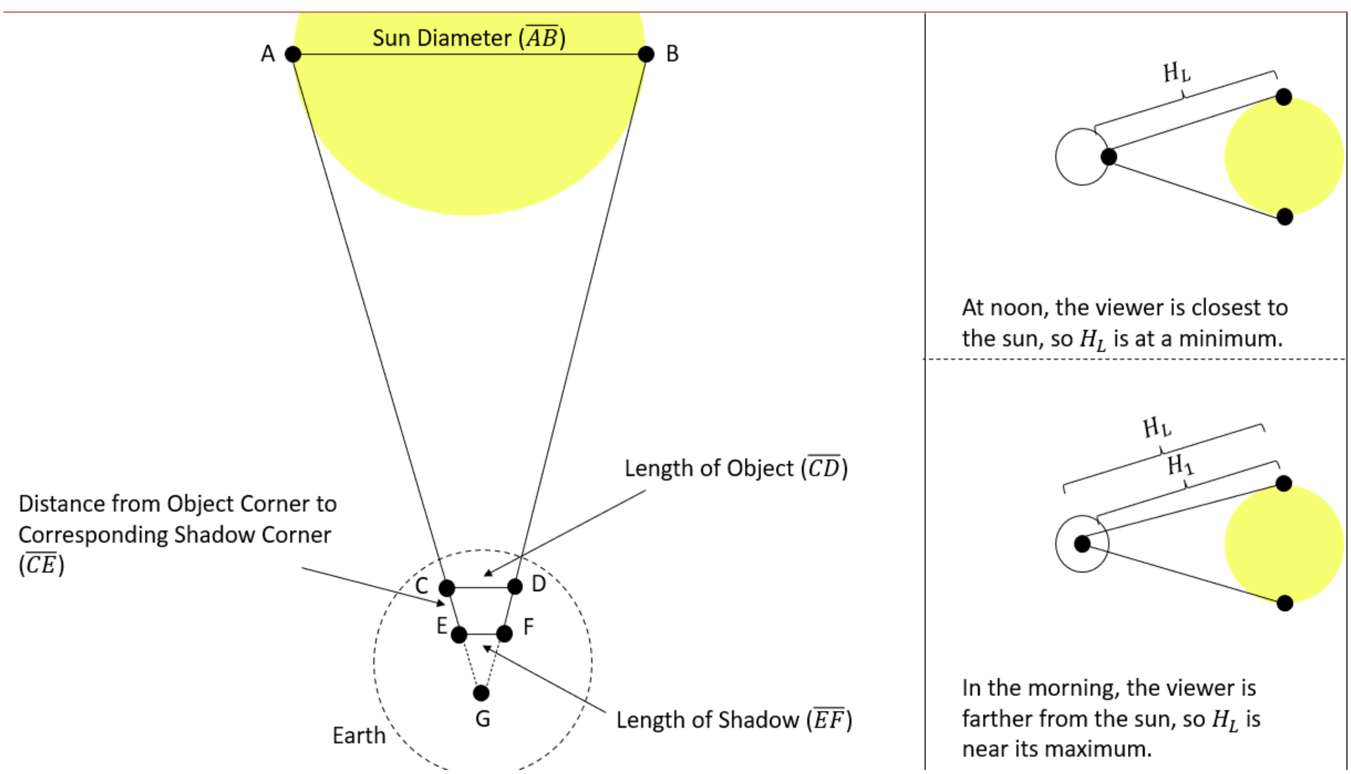

Figure 2. Diagram of the Sun, Earth, and necessary measurements for the shadow method. The line segments, values, and triangles in this diagram are used in the derivation of the new geometric formula used in the shadow method. The right side depicts the change in calculated Sun-Earth distance over time of day. Not to scale.

edge, not its center. However, calculating $\overline{AE}$ is faster than calculating the distance from the Earth to the center of the Sun because calculating $\overline{AE}$ requires measuring $\overline{CE}$ (distance from shadow corner to object corner), while measuring the true distance between the Earth and the Sun requires measuring the distance from the center of the shadow to the center of the object. $\overline{CE}$ is easier to measure, as the precise centers take longer to accurately identify than the corners. Therefore, calculating $\overline{AE}$ is easier and removes unnecessary complications. Furthermore, as the distance between the Earth and the Sun is more than 100 times greater than the diameter of the Sun, for the purpose of the experiment in our study, the discrepancy between $\overline{AE}$ and the true Sun-Earth distance is statistically insignificant, especially for the casual observer, which is the primary focus of our study, and the hypothesis remains valid regardless (5). It is also worthwhile to note that while the time of day is controlled for by having all shadow and pinhole methods tested throughout the day, the variation in calculated Sun-Earth distance over the time of day should only change by at most the Earth's radius, which is extremely insignificant. As long as the sun is visible, the only change in the calculated distance will be that caused by the Earth's rotation, which reflects the changing distance of the viewer and the Sun over the course of a day (**Figure 2**).

MATERIALS AND METHODS

The derivation for the shadow formula, which is used for the shadow method, is as follows (refer to **Figure 2** for symbols):

Triangles ΔABG, ΔCDG, and ΔEFG are all similar due to angle-angle similarity (they share $\angle G$ and since $\overline{CD}$, $\overline{EF}$, $\overline{AB}$ are all parallel, $\angle EFG = \angle ABG = \angle CDG$).

$$\frac{\overline{EG} + \overline{CE}}{\overline{EG}} = \frac{\overline{CD}}{\overline{EF}}$$

$$\frac{\overline{CE}}{\overline{EG}} + 1 = \frac{\overline{CD}}{\overline{EF}}$$

$$\frac{\overline{CE}}{\overline{EG}} = \frac{\overline{CD}}{\overline{EF}} - 1$$

$$\overline{EG} = \overline{CE} * (\frac{\overline{CD}}{\overline{EF}} - 1)^{-1}$$

$$\overline{EG} = \overline{CE} * (\frac{\overline{CD} - \overline{EF}}{\overline{EF}})^{-1}$$

$$\overline{EG} = \frac{\overline{EF} * \overline{CE}}{\overline{CD} - \overline{EF}}$$

The first step is obtaining the value of $\overline{EG}$:

$$\frac{\overline{AE} + \overline{EG}}{\overline{EG}} = \frac{\overline{AB}}{\overline{EF}}$$

$$\overline{AE} + \overline{EG} = \frac{\overline{AB}}{\overline{EF}} * \overline{EG}$$

$$\overline{AE} = \frac{\overline{AB}}{\overline{EF}} * \overline{EG} - \overline{EG}$$

$$\overline{AE} = \overline{EG} * (\frac{\overline{AB}}{\overline{EF}} - 1)$$

The next step is to obtain the value of $\overline{AE}$:

The next step is to substitute $\overline{EG}$ with the value calculated

$$\overline{AE} = \frac{\overline{EF} * \overline{CE}}{\overline{CE} - \overline{EF}} * (\frac{\overline{AB}}{\overline{EF}} - 1)$$

in the first step:

Finally, values can be renamed to common names (as

$$\overline{AE} = \frac{Length\ of\ Shadow * Shadow\ Corner\ to\ Obj\ Corner}{Length\ of\ Obj - Length\ of\ Shadow} * (\frac{Sun\ Diameter}{Length\ of\ Shadow} - 1)$$

seen in **Figure 2**) to obtain the final formula below:

To obtain the necessary measurements for distance calculations, card measuring under-1-mm-thick 9.3 x 13.2 cm was used. The measurements were taken from morning to evening within one day. For the measurements before noon (Samples 1-5) the card was placed on a 58 cm high windowsill, taped down, in front of an East-facing window. The card's longer edge was protruding a few cm off of the windowsill so that the protruding edge casted a shadow. For all other samples excluding Sample 7, the paper was placed on the edge of the 47 cm high seat of a wooden chair, indoors, in a similar manner to the previous samples. The chair was in front of a West-facing window, so that the Sun cast a shadow of the protruding edge of the paper. For Sample 7, the card was placed on a wooden chair outside in a similar manner to all other samples. In all samples, the Sun was unobstructed in the sky and the paper and card received direct sunlight. There were no indoor lights turned on and the shadows were clear, with no blurriness at their edges.

A standard wooden 12-in ruler and string were used for measuring. The lengths of the shadows were always measured first, and then the corner measurements were obtained (**Figure 3**). Prior to measuring each sample, a flat object (i.e., a sheet of white paper) was placed under the shadow to improve clarity of the shadow, and the ruler was placed adjacent to the shadow edge corresponding to the object edge protruding off the raised surface. After the shadow length was measured and recorded, the corner measurement was made in a matter of seconds following the shadow measurement in order to minimize the risk of the measurements changing as the Sun moved across the sky. To measure the distance between the shadow corner and the corresponding object corner, a thin string was held by a finger at the bottom-left (from the perspective of the observer opposite the window and in-front of the protruding edge of the object) corner of the shadow and drawn up to the corresponding bottom-left corner of the object. The string was then cut at the places where they met the corners and measured and recorded. The object was taken from the raised surface, placed on a flat wooden table, and had its length measured. All measurements were taken to the nearest mm. These measurements were then substituted into the newly derived shadow formula (along with the Sun diameter value of 1.397×10^{11} cm) to calculate the distance from the object to the Sun. To find the error values, the calculated distances

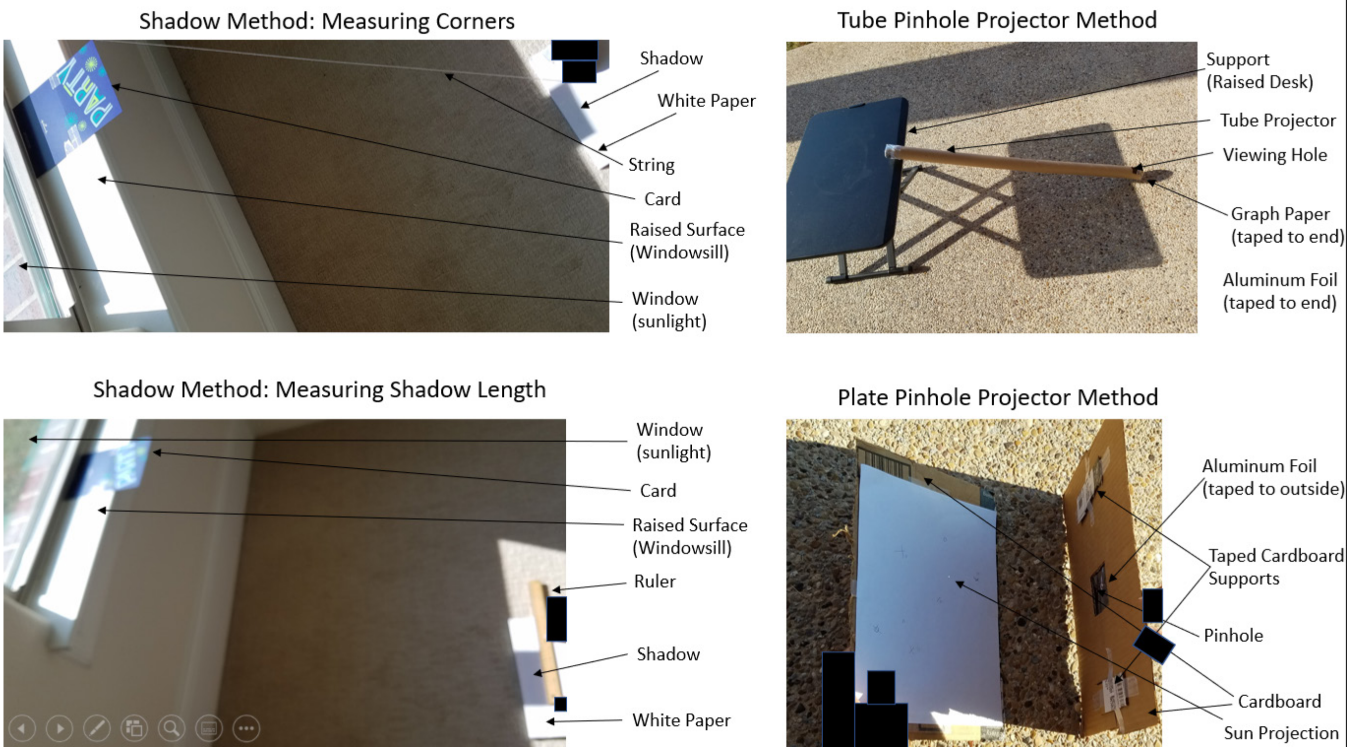

Figure 3. Photos of implementations for tube pinhole method, plate pinhole method and shadow method. The photos depict the implementation of each method. Labels are used to identify important parts of the implementations.

were compared with the distance between the Earth and Sun during Aphelion Day 2021 on July 5th, which is 1.521×10^{13} cm (7). Since the shadow measurements were done in early July, the official distance on this day is a good approximate of the true distance between the Earth and Sun during the day in which the experiment was done.

To utilize the plate pinhole method, a set of different steps had to be followed to measure the samples as done in **Table 2**. The procedures for this method were based off those in Source 5 (5). First, a pinhole projector was constructed using a 2-mm thick, 28.2 x 33 cm panel of cardboard. A 7.2 x 4.8 cm rectangular hole was cut in the middle of the panel with a boxcutter. A 12 x 10.8 cm piece of aluminum foil was attached over the hole and was taped down at the edges. A safety pin was poked through the center of the aluminum foil to form a pinhole which looked to have a ¾ mm diameter. On the underside of the projector (where the foil is seen through the rectangular hole), a red circle was drawn around the pinhole using a marker (to make locating the hole easier). To build the surface upon where the Sun would be projected, a sheet of standard computer paper was taped onto a 2-mm thick, 22 x 33.3 cm cardboard panel. For the measurements before noon (Samples 1-3) the projector and projection surface were placed outdoors, facing East. For all others, the materials were set up either indoors or outdoors (depending on the availability of the Sun) facing West. The topside of the projector (the side where the aluminum foil is taped down) was positioned facing the Sun, and both the projector and surface were angled at various degrees and were a certain distance apart so as to allow the Sun to project onto the projection surface (**Figures 3, 4**). These angles and distance were not measured in order to conform with conventional procedures, as they were not regarded as important in conventional pinhole method procedures supported by NASA (5). For the same reason, as done in NASA procedures, the projector and surface were held up with hands (5). Like the shadow method measurements, in all samples, the Sun was unobstructed in the sky and the paper and card received direct sunlight

In measuring each sample, a standard 12-in ruler, string, and pencil were used. First, a circle was drawn using a pencil on the projection surface surrounding the Sun projection. Second, a length of string was stretched between the Sun projection and the pinhole and was then cut appropriately to reflect the distance between the two. The diameter of the drawn circle was then measured, as was the length of string. All measurements done were taken to the nearest mm. These measurements were then substituted into the pinhole formula obtained from the NASA supported procedures (along with a sun diameter value of 1.397×10^{11} cm) to calculate the distance from the projector to the Sun. These measurements were done in late July, therefore, to calculate the error values, we believed it more appropriate to use the average Sun-Earth distance (1.496×10^{13} cm) instead of that on Aphelion Day 2021, as done for the shadow method (5).

The procedures to utilize the tube pinhole method were based off those in Source 4 (4). A 101.5 cm hollow cardboard

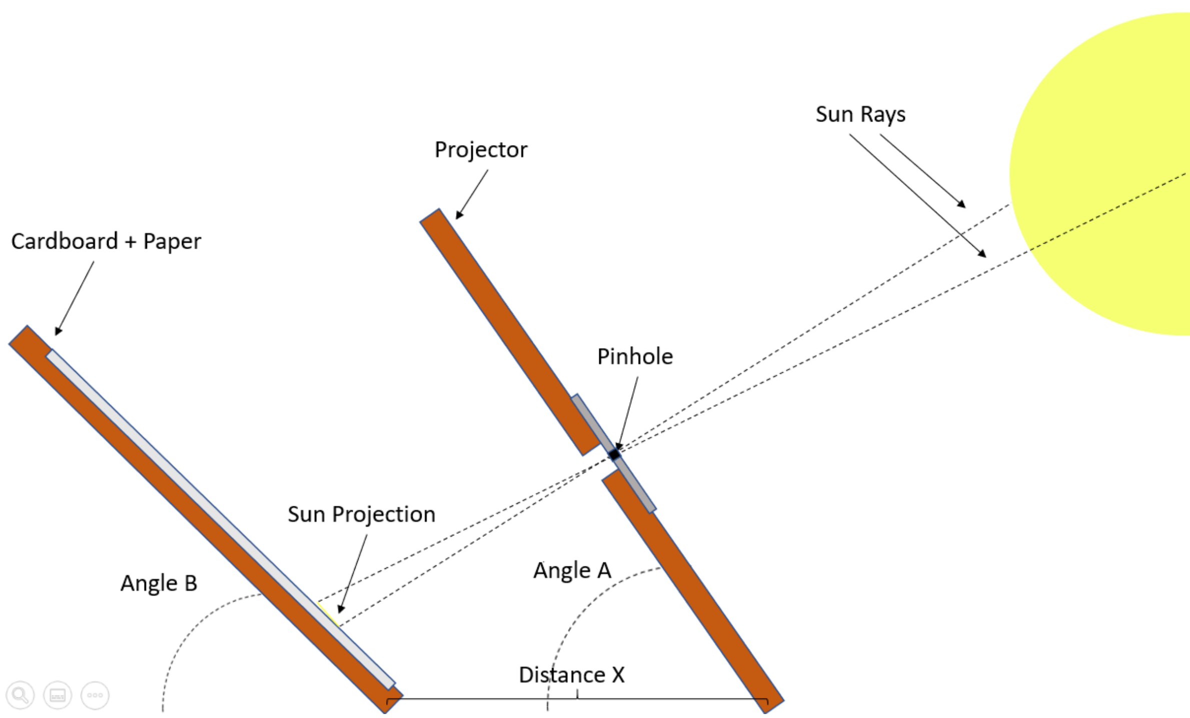

Figure 4. Diagram of the plate pinhole projector and projection surface. The figures in this diagram are the core materials used in the pinhole method.

tube served as the base, with a piece of aluminum foil and a circular piece of mm-spaced graph paper taped down at either end of the tube, covering up the holes. A boxcutter was used to cut a 5 x 4 cm rectangular hole into the side of the tube, 1 cm from the hole covered by the graph paper. A safety pin was used to poke a pinhole of what looked like 0.75 mm, but when a projection didn't manifest during observation, it was slightly widened to 1mm. Eight times throughout a day in late September, the tube projector was placed outside, oriented toward the Sun so that an image of the Sun manifested on the graph paper, and was propped up against an adjustable desk (**Figure 3**). While the projector typically rested completely on the desk, in Samples 4 and 5, the steep angle required for the projector when the Sun was high required it to be held by a hand as well. The diameter of the casted image was determined by counting 1mm lines on the graph paper, seen through the viewing hole. This measurement, the time, and the predetermined length of the tube was recorded and input into the pinhole formula used with the plate pinhole method (along with a Sun diameter value of 1.397×10^{11} cm) to calculate the distance from the projector to the Sun. As with the plate pinhole method, the average Sun-Earth distance (1.496×10^{13} cm) was used to evaluate accuracy.

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