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Article

A study on the stretching behavior of rubber bands

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SUMMARY

Rubber bands are simple and familiar household items whose properties are worth researching to determine their utility and value. They are used ubiquitously, as toys, binders for objects and as equipment in the fitness industry, which necessitates study of their properties and manufacturing and labelling them based on standard extensibility and strength, specific to utility. Our study aimed to investigate the applicability of Hooke's law to rubber bands. We hypothesized that rubber bands are ideal elastic materials, exhibiting stretch in direct proportion to loads and that bands of any length respond equally to pulling forces. We exposed a few rubber bands to a step-wise longitudinal loading process, and the stretch response after each step of the load was measured. While the initial loading steps removed only the slack of bands, a marked stretch behavior was observed soon. A linear relationship between stretch and applied force, noted in a few intermediate steps, was lost eventually. When we compared the bands of different lengths, it was obvious that the shorter bands stretched less than the longer ones for similar loading. Finally, we rejected the hypothesis since the findings supported that rubber bands do not follow Hooke's law and that their initial lengths affect the stretch response for loading. This study can be used in further research of the properties and molecular structure of rubber bands and to compare them with some visco-elastic structures.

INTRODUCTION

Elastic substances respond to forces acting on them by showing a change in their dimensions and return to their size when force is withdrawn from them. Stress is the force imposed on the unit area of the substance, and strain is the ratio of change in size to the original size (1). Hooke's law states that strain is directly proportional to the stress subjected within a specific range on an elastic substance, and the constant for proportionality is called the elastic modulus. Thus, according to Hooke's law, $F = -k\Delta x$, where: F is the spring force, k is the elastic modulus specific to the material, and Δx is the displacement (1).

In daily life, rubber bands are familiar objects widely used to bind objects together. However, they are also used in various other applications such as 'do-it-yourselves' toys and the fitness industry. If rubber bands behave ideally like elastic objects, and recoil to the original shape after use, their applications and durability will differ from those of non-ideal elastic objects. Establishing the properties of rubber bands and determining their specifications would be crucial to classify, grade, and improve their standards for future use. For instance, rubber that can withstand pulling forces can be used as therabands in the fitness industry, whereas that having has high hysteresis, the elastic energy dissipated per unit volume during the deformation process, is suitable for manufacturing sound and vibration absorption materials. Hysteresis can further be understood as the difference of energy between the elongation and the contraction of the rubber bands and is presented as the area between the loading and unloading part of a graph. Similarly, ideal elastic rubber bands, if available, can be used to measure weights of objects because they will have a property to stretch in proportion to forces applied, like spring gauges do.

Researchers debate Hooke's law's applicability to rubber bands to consider if they can be used as measuring tools (2,3). While some studies claimed that rubber bands followed a linear stretching pattern in response to stress, other researchers concluded that Hooke's law is not applicable to rubber bands as they showed non-linear response, rendering to its stress-dependent response (3). Some researchers reported that bands showed decreasing slope, followed by a constant slope, and then a third part with a rising slope when stress and corresponding strain were plotted (4). Studies on length characteristics reported that longer bands extended to a greater extent than the shorter ones (5).

We had earlier been critical of studies which claimed that rubber bands were not ideally elastic. When we pulled the ends apart, they extended well, but this was applicable to single instance, short-lived pulls. The bands that were used to tie the hair or, the therabands for exercises and the bands of the catapult (toy) gave more insight, showing stiffening after pulling to some extent and also deformation on multiple use. We felt that the rate and amount of load and type of rubber bands influenced stretch response and that using these factors in experimenting could elicit the extensibility properties of bands in an elaborate form. Many researchers studied the stretching of rubber bands, but the observations contradicted each other to some extent. Hence, starting with our general notion, we hypothesized that rubber bands stretch proportionally to longitudinal loading and then recoil in the same path when stress is relieved, thus behaving as ideal elastic or Hookean materials. Furthermore, we also hypothesized that the bands show similar responses at any length. However, the results were surprisingly contrary to what we assumed, and our study culminated with the rejection of the hypothesis. Rubber bands did not stretch in proportion to their loading, and they recoiled in a different path when the load was removed. In addition, it was noted that the stretch response was affected by the length of bands.

RESULTS

The Effects of Step-Wise Loading on Rubber Bands

We used three unstretched rubber bands of 6 cm length for the experiment. Longitudinal loading of one rubber band was done in 12 steps of 36 g (4 coins of 5 rupees in Indian currency) in each step, and the stretching of the band was noted. The first reading was recorded only with the empty loading container of 36 g, which removed the slack in the rubber band. Stretching of rubber bands was studied by loading them with standard weights of household objects like coins, that have standard weights (6,7). Each step after that added 36 g, till the total load at the 12th step was 432 g (44 coins + weight of container). Upon each step of loading, the rubber band stretched. We recorded the responses of two more bands in a similar way, and calculated the mean of the three readings at each step. The response of the rubber bands to the various loads was noted **(Table 1)**. Next, we calculated the forces by the formula F=mg, where 'F' is the force in Newton, 'm' is the load converted to kilogram and 'g' is the acceleration due to gravity (taken as $10 m/s^2$).

The initial loading steps increased the length only negligibly to obliterate the slackness of the bands, but the graph quickly showed marked stretch responses till the bands gained 10.3 cm at 180 g load (170%) **(Table 1)**. The bands showed a stretch of 20.56 cm, amounting to a total 340% elongation with 288 g loading. The later steps showed a decrease in stretch behavior with only a 6.8 cm stretch between 288 g to 396 g loading. The bands stretched about 31.9 cm (530% of the original length) by the end of the 12^{th} step. Upon unloading in a step-wise manner, the rubber bands showed contraction with each unloading step but did not return to their corresponding loading step length. We were surprised to note a gain of 12 cm in length, reflecting 100% permanent deformation at the end.

Next, we plotted a graph with stretch or displacement readings on the X-axis and force on the Y-axis **(Figure 1)**.

Step No.	Load (g)	Force (Newton)	Stretch (cm)				
			Trial 1	Trial 2	Trial 3	Mean of 3 trials of stretch (cm)	Std. deviation
1	36	0.36	0. 1	0. 1	0. 1	0. 1	1.69
2	72	0.72	1.5	1.5	1.5	1.5	0
3	108	1.08	3.2	3.3	3.4	3.3	0.1
4	144	1.44	6.5	6.4	6.4	6.43	0.057
5	180	1.8	10.2	10.4	10.3	10.3	0.1
6	216	2.16	14.5	14.9	14.9	14.76	0.230
7	252	2.52	17.6	17.7	17.9	17.73	0.152
8	288	2.88	20.2	20.8	20.7	20.56	0.032
9	324	3.24	22.7	23	23.3	23	0.3
10	360	3.6	24.5	25	25.6	25.03	0.55
11	396	3.96	27.2	27.4	27.5	27.36	0.152
12	432	4.32	32.4	31.9	31.4	31.9	0.5
Unload	396	3.96	32.3	31.9	31.4	31.86	0.450
Unload	360	3.6	31.9	31.8	31.2	31.63	0.378
Unload	324	3.24	31.5	31.1	31	31.2	0.264
Unload	288	2.88	31	30.9	30.6	30.83	0.208
Unload	252	2.52	30.2	30.1	30.3	30.2	0.1
Unload	216	2.16	29.8	29.4	29.6	29.6	0.2
Unload	180	1.8	28.3	28.2	28.5	28.33	0.152
Unload	144	1.44	27.1	27	27.3	27.13	0.152
Unload	108	1.08	26	25.9	25.6	25.83	0.208
Unload	72	0.72	24	23.5	23.4	23.63	0.321
Unload	36	0.36	18	17.8	18	17.93	0.115
Unload	0	0	6	5.8	5.5	5.76	0.251

Table 1. Stretch (cm) of the 6 cm rubber bands during the 12-step loading and unloading.

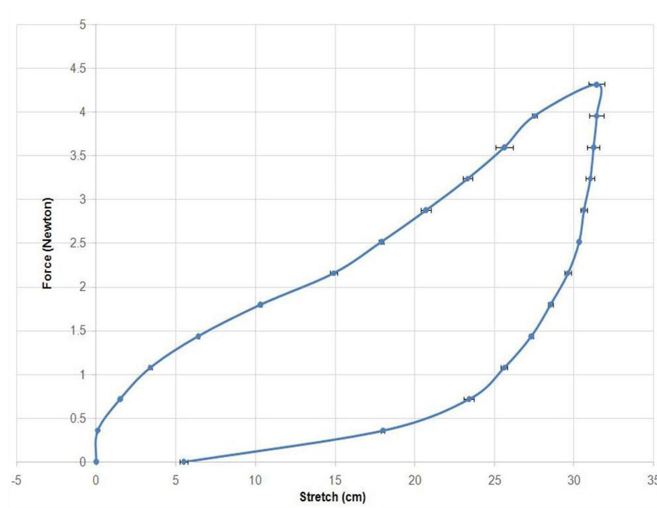

Figure 1: 12-Step loading and unloading of 6 cm bands. Force (in Newtons) vs. mean stretch (in cm) plot depicts the mean stretch in three 6 cm bands subjected to stepwise loading. The error bars represent the standard deviation. The upper line and lower line represent responses to loading and unloading, respectively. Linearity is presented between steps 6-7 and 7-8. A mean stretch of 31.9 cm (530% of the original length) was noted during loading with 432 g. The hysteresis loop is evident, indicated by two different lines for loading and unloading.

Steps	Slope (in N/cm)
1 - 2	0.25
2 - 3	0.2
3 - 4	0.11
4 - 5	0.09
5- 6	0.08
6 - 7	0.12
7- 8	0.12
8- 9	0.14
9- 10	0.17
10- 11	0.15
11- 12	0.07

Table 2: Slopes for the 12-step loading of the 6 cm rubber bands (Force vs. Stretch)

Step No	Load (g)	Force (N)	Mean (6 cm band stretch in cm)	Std. deviation	Mean (3 cm band stretch in cm)	Std. deviation
1	108	1.08	2.33	0	0.96	0.057
2	216	2.16	12.1	0.15	4.76	0.057
3	324	3.24	20.3	0.1	7.46	0.152
4	432	4.32	27.1	0.2	9.76	0.152
Unload	324	3.24	25.3	0.173	9.2	0.1
Unload	216	2.16	21.3	0.264	7.8	0
Unload	108	1.08	14.8	0.173	5.06	0.057
Unload	0	0	4.5	0.1	0.90	0.005

Table 3: Mean stretch (in cm) in 6 cm and 3 cm bands during loading with 432 g.

The standard deviations of the displacement were used to determine the error bars in the graph. Though the visual appearance of the graph **(Figure 1)** shows a rough linear appearance, the non-linear elongation of the rubber band was identified by slopes **(Table 2)**. The slope remained constant in the intermediate steps, *i.e.*, 6 to 7 and 7 to 8 (slope of 0.12 N/cm) of the experiment, indicating a linear response. The bands stretched between 14.76 cm to 20.56 cm in this region. However, this trend was soon lost. An increment from 240% to 340% in length was found before the slope significantly increased, as seen in the graph. The slope of the graph where (x1, y1) are coordinates of the first point and (x2, y2) are coordinates of the second point at each step, with the variable of displacement (stretch) and force on x and y coordinates, respectively is calculated as m(slope)=(y2-y1)/(x2-x1).

Studying Stretch with Varying Lengths of Rubber Band

We used three rubber bands of 6 cm length and another three of 3 cm length of the same type and thickness for this part of the study. Loading of 432 g was done in four steps for each rubber band individually, and the bands were not allowed to slacken between the steps. The stretch of the rubber bands was noted in both cases by the displacement at the distal end. The step size was 108 g, and the total weight by the fourth step was 432 g. An average of the responses of three rubber bands of each length was used for further observation. The mean stretch in the three bands of 6 cm length was 2.33 cm for a load of 108 g, which increased swiftly with addition of another step. We observed that at the end of the fourth loading step, the bands extended 27.1 cm. The 3 cm rubber bands on the other hand, showed 0.96 cm mean stretch with the first step loading of 108 g, and attained a final stretch of 9.76 cm at the end of the fourth step **(Table 3)**.

The results showed that when the rubber bands of 6 cm and 3 cm length were subjected to the same load of 432 g, the 3 cm rubber band showed lesser stretch (9.76 cm amounting to 325% of original length) than the 6 cm rubber band (27.1 cm amounting to 450% of original length). Standard deviations are also presented in the table **(Table 3)**. A graph was then

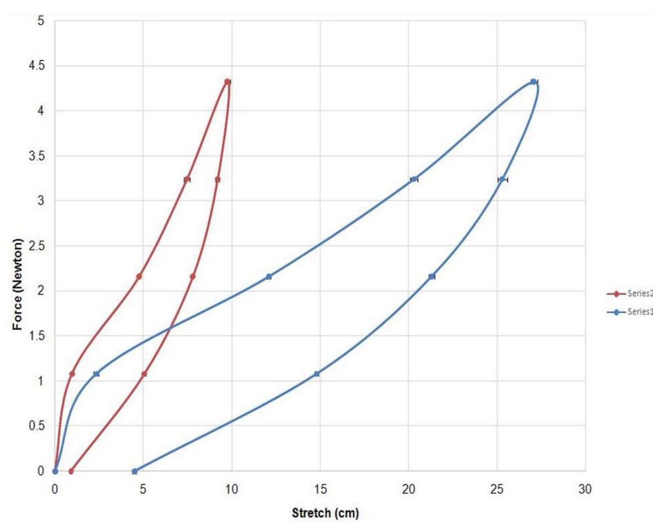

Figure 2: 4-Step loading and unloading of 6 cm and 3 cm bands. Force (in Newtons) vs. mean stretch (in cm) plot shows the mean stretch in three bands each of 6 cm (blue) and 3 cm (red) when subjected to stepwise loading and unloading. The upper line and lower line for each type of band represent responses to loading and unloading, respectively. The error bars represent the standard deviation. The 3 cm rubber bands and the 6 cm rubber bands showed stretches of 9.76 cm (325%) and 27.1 cm (450%) during loading with 432 g. The loading and unloading responses followed different paths for each band, indicating hysteresis.

plotted with the mean stretch for the load applied and the standard deviations are indicated by error bars for the bands of both lengths **(Figure 2)**. During unloading, the recoil in both bands was in a way that the bands' length remained more than the corresponding loading step. Also, we noted that the shorter bands showed less permanent deformation than the larger ones (0.9 cm, 4.5 cm) at the end of the experiment. The stretch response for the bands can be compared pictorially **(Figure 2)**. It was also evident that the 12-step loading caused a high stretch compared to any given step of the 4-step loading **(Tables 1 and 3)**.

DISCUSSION

Initially, during the loading phase of the 6 cm rubber bands, the increase in length compared to the original length, also called stretch or displacement, was minimal and removed the slack of the bands. After the slack disappeared, the slope decreased, showing increased stretch with loading. Then, a roughly linear relation between force and stretch was observed for some steps of loading till a length of 20.56 cm was gained. However, this linear pattern started deviating later. An increment of about 240% to 340% in length was found before the slope significantly increased **(Figure 1)**. The rubber bands showed lesser elongation with each step henceforth. This loss of ability to further change its length is called stiffness. Hence, rubber bands show an eccentric behavior to loading, based on their history of load exposure, portraying non-linear stretch response for load exerted each time. Thus, the stretching of the rubber bands is load-dependent, and rubber bands are not ideal elastic materials as the unloading curve and the loading curve did not follow the same path (3,8). The rubber bands did not recoil into the corresponding loading stage at each step during the unloading phase. The graph showed a diverging path from the loading path when the load was removed, creating a loop-like structure overall. This loop shows that the rubber band material has elastic energy at any point different from stress-energy, which is the magnitude of the externally applied force per unit area of the material used to create the elongation (8). The results of this study confirm previous research showing the force-extension curve of rubber bands depicts significant deviation for Hooke's law at higher levels of stretching (9). Finally, after completing the experiment, the rubber bands were observed to be permanently deformed by 100% of the initial 6 cm. To summarize, the stress-strain curves of rubber bands showed one region of decreasing slope, followed by an almost a constant slope, and a third part with a rapid rising slope.

We demonstrated that the length factor of rubber bands impacted the extent of stretch. Shorter rubber bands stretched less and recoiled more for the same unloading compared to the longer ones. When a 432 g was applied in four equal steps, the 3 cm and 6 cm bands stretched about 300% and 450% of their lengths, respectively. At the end of the experiment, after rubber bands were detached from the system, the permanent deformation was 0.9 cm (33%) and 4.5 cm (75%) for the 3 cm and 6 cm rubber bands, respectively **(Figures 1 and 2)**. It was hence inferred that longer bands stretch more than the shorter ones. Also, it was observed that the velocity of loading influences stretch, with 12-step loading causing more stretching than 4-step loading. The reason for this increase in stretching may be that the constant slow force in 12-step loading causes a stretch to certain distance that further increases with time as the band relaxes (10).

This study led to some significant conclusions. For both parts of the experiment, Hooke's law, written as $F = -k\Delta x$ or $K=g(dm/dx)$ with the positive coordinate downward, is supported only in a part of the experiment. The hypothesis that rubber bands showed linear stretching response upon longitudinal loading was rejected because they showed a non-linear response overall. Next, the hypothesis that the initial length of the rubber band showed no effect on stretching was rejected because the observations showed that the longer rubber bands stretched more. Furthermore, the hypothesis that the rubber bands recoiled fully during unloading was also rejected as recoiling took a different path than stretching during loading. Therefore, our study supports that rubber bands do not follow Hooke's law and have unique properties based on their structure.

This study can be extended to evaluate the area between the loading and unloading lines of the graph for understanding hysteresis. The differences between multiple-step loading and single-step loading and the impact of factors like thickness, temperatures, and brands can also be studied. The probable limitation of this study may be the errors due to a looped knot at the connecting points. However, to minimize errors, we ensured that the bands had no twists or loops in length between the connections. A looped knot is justified at the connection, because if the bands were simply clamped with available clips at home, they quickly slipped from the clips with minimal loading. A simple knot was avoided because it would inevitably pull and stretch the rubber bands before the experiment, impacting the readings. On the contrary, the loop automatically fastens the rubber band (without pulling while fastening) while loading, leading to better reliability.

The property of elongation and recoil of rubber is of particular importance to professionals like physical therapists, who use therabands and exercise tubing for therapeutic reasons. This study may have implications for the durability and maintenance of such items. Rubber bands showed load-dependent responses to forces. Hence, this study of rubber band properties may offer inputs to the studies on visco-elastic structures in the body.

MATERIALS AND METHODS

We constructed an experimental set-up using domestic objects and basic measurement systems to study the stretch behavior of rubber bands upon loading **(Figure 3)**. New rubber bands of the same lot and dimensions were taken for each part of the experiment.

Figure 3: Picture of the experimental setup. A rubber band is fixed to one end with a hook and to the other with a plastic container for loading the system with weights. A scale was arranged adjacent to the system to measure stretch.

We suspended rubber bands from a hook and subjected them to loading in various forms to study their stretch response. The upper end of the rubber band was fixed to a hook by a looped knot. The lower end of the rubber band was attached to the container using a binder clip and embedding a toothpick that would act as a pointer in measurements. A plastic container that just removed the slack of the rubber band was used for the experiment, and its weight was 36 g. The loading container was suspended in the air without rubbing against any surfaces. 5 rupee coins in Indian currency (standard weight 9 g) were used for the experiment

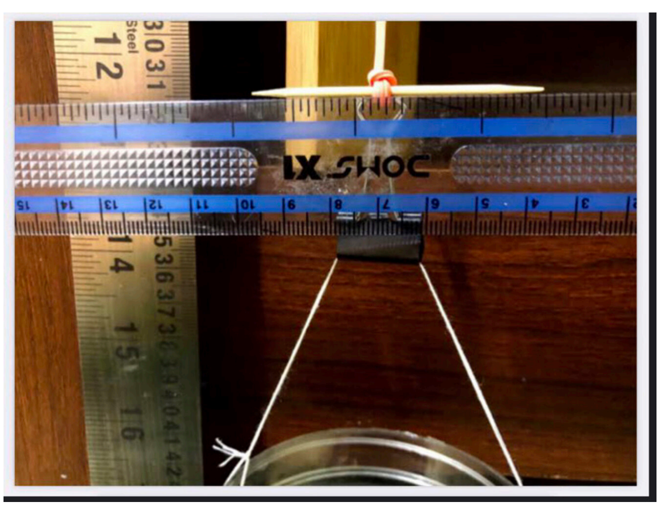

Figure 4: Picture of stretch measurement. A meter scale with cm and mm marked on it was arranged adjacent to the rubber band. A toothpick embedded at the junction of the weight and the rubber band's distal end was used as a pointer for measurement of stretch (displacement of the distal end of the rubber band) on the scale.

for loading. We calculated the forces by the formula F=mg, where 'F' is the force in Newton, 'm' is the load converted to kilogram and the 'g' is the acceleration due to gravity (taken as 10 m/s^2).

A rest time of three minutes was given between each step without slackening the bands. A meter scale with cm and mm marked on it was fixed just adjacent to the rubber band so that the toothpick pointed on its marks **(Figure 4)**. The length between final and initial marking at each step, as pointed by the toothpick on the scale, was recorded as stretch. A stop clock on a mobile phone was used for time measurement during the study. Graphs for all the stretch responses in relation to the forces applied are plotted using Microsoft EXCEL.

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