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Article

Motion tracking and analysis of spray water droplets studied by high-speed photography using an iPhone X

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SUMMARY

Smartphones are not only becoming an inseparable part of our daily lives, but also a low-cost, powerful optical imaging tool for more and more scientific research applications. In this work, we hypothesized that some smartphones could be used as a low-cost, high-speed, photographic alternative to expensive equipment, such as those typically found in scientific research labs, to accurately perform motion tracking and analysis of fast-moving objects. This demonstrative experiment was done at home at very low cost by using a low-power green laser (<0.1 W) purchased from Amazon, an iPhone X, and a water spray bottle. The sprayed water droplets were illuminated by a divergent green laser beam and recorded with a smartphone using the Slow-Motion Video function (240 frames per second), which allows us to record high-speed events. The Slow-Motion Video was converted to a series of consecutive images taken every 1/240 second (or 4.16 ms), enabling tracking and motion analysis of the sprayed water droplets. By analyzing these consecutive images, the speed and flight trajectory of water droplets in the air were obtained, thereby enabling us to estimate the area of the water droplets landing on the ground.

INTRODUCTION

Due to the rapid progress of semiconductor hardware and software technology in recent years, smartphones have increased in power dramatically with no concomitant increases in cost, making them very affordable and highly capable devices. Nowadays, smartphones have become an inseparable part of our daily lives enabling individuals to use a single device for multiple functions – making phone calls, browsing the internet and social media, playing music, setting alarms, as well as an imaging tool for scientific research applications, particularly for medical and biological applications. For example, a smartphone was used to replace a bulky optical microscope to perform the diagnosis of hematologic and infectious diseases (1). Additionally, they have been used as a wide-field fluorescence microscope for fluorescent imaging of labeled white-blood cells from whole blood samples illuminated with battery powered blue-green light-emitting diodes (2). A smartphone was also transformed into an endoscope for acquiring otorhinoscopic images from patients for remote diagnosis of ear and nose diseases (3). In addition to medical and biological applications, smartphones have also been used for chemical (4), physical (5), and optical measurements (6).

Recently, scientists at the National Institutes of Health (NIH) used a smartphone to visualize the emission of aerosols and droplets when a person was speaking (7). During speech, the number of droplets emitted from the person's mouth was measured as a function of time by analyzing video clips of the speech events, which were illuminated by a bright light sheet generated from a high-power green laser (at 2.5 W output power). The laser-based light sheet could selectively illuminate those droplets populated in one plane, eliminating image interference introduced by the droplets that were out of the plane. The video (8), recorded with a smartphone at 60 frames per second (fps), provided the public with very good educational material about the biological impact of droplets emitted from a person's mouth. However, the authors did not study motion characterization of the droplets.

Inspired by this recent work (7) and given the fact that the iPhone X can provide video camera recording at 240 frames per second (240 fps), we hypothesized that the iPhone X could also be used as a low-cost high-speed photographic tool to accurately perform motion tracking and analysis of fast-moving objects. To test this hypothesis, a low-cost demonstration experiment was done at home using a low-power green laser (<$100, <0.1 W output power) purchased from Amazon, an iPhone X, and a water spray bottle to quantitatively investigate the projectile motion of droplets in the air. The water droplets were illuminated by a divergent green laser beam and recorded with the Slow-Motion Video function (240 fps), allowing for high-speed photography. The Slow-Motion Video was then converted to a series of consecutive images taken every 1/240 s (or 4.16 ms) (9), enabling the tracking and motion analysis of the water droplets.

By analyzing six consecutive images obtained from a segment of slow-motion video at the beginning of a water spray process, the trajectories of six individual specific droplets were identified, and their travelling distances and speeds were obtained. Additionally, a horizontally launched projectile motion of one specific water droplet was captured in a segment of slow-motion video at the end of a water spray process, enabling us to measure its vertical acceleration to be 985 mm/s^2. We used a simple projectile landing formula to calculate droplet trajectories and estimated the size of the circumferential area where the sprayed water droplets could be landing on the ground. Our experiments demonstrated that a smartphone can be a low-cost tool to quantitatively investigate projectile motion of droplets in the air.

RESULTS

A strong stream of water droplets can be generated from the nozzle of the spray bottle immediately after squeezing the pump dispenser. By observing the whole process with the Slow-Motion Video function of an iPhone X camera, we found that water droplets initially come out from the spray bottle in a very small density, which allows us to track and analyze trajectories of several individual droplets. Then, the water droplet populations quickly become denser while increasing in speed. In the middle of the water-spraying process, the density is so high that no clear image of individual water droplets could be captured (**Figure 1**). As all the water droplets have exited, the opposite behavior occurs where fewer water droplets come out, their travelling speeds decrease over time, and they eventually stop.

Spray angle and travelling speed

The spray angle is depicted by the dashed line within the boxed area and was measured to be 60 degrees from the image plane (**Figure 1**). Approximately, we can assume the spatial distribution of spray water droplets is radially symmetrical, as the shape of the spray nozzle. If this assumption is correct, then it means the water droplets are coming out in a corn shape with an apex angle of 60 degrees.

The trajectories of individual water droplets can clearly be seen from several of the opening images taken. We extracted six images (**Figure 2**) from a piece of the slow-motion video depicting the initial spray phase by using online software (Free Video to JPG Converter) (9). The frame rate of the Slow-Motion

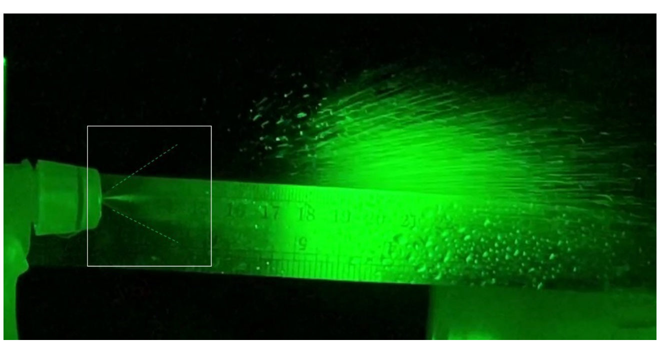

Figure 1. A typical snapshot image taken in the middle of a water-spraying process, with a spray angle depicted by the dashed line within the boxed area.

Table 1. Motion tracking and analysis of 6 individual droplets in both a pixel coordinate system and the physical unit coordinate system.

Droplet	Start point (pixel coordinate)	End point (pixel coordinate)	Distance (pixels)	Physical distance (mm)	Speed (m/s)
1	(351, 167)	(441, 229)	109.3	19.9	4.8
2	(427, 176)	(556, 136)	135.1	24.6	5.9
3	(402, 201)	(518, 175)	118.9	21.6	5.2
4	(426, 221)	(532, 197)	108.7	19.8	4.7
5	(644, 240)	(743, 233)	99.2	18.0	4.3
6	(699, 251)	(813, 249)	114.0	20.7	5.0

Video used in the experiment was 240 fps. In other words, the time interval between these consecutive images is 1/240 second or 4.16 ms. The trajectories of six specific individual droplets were identified and marked as droplet “1”, “2”, “3”, or “4” travelling in one frame period (1/240 s) (**Figures 2D, 2E**) and droplet “5” and “6” travelling in another frame period (**Figures 2E, 2F**). We defined the location of each individual droplet in a pixel coordinate system from each frame (**Table 1**). The travelling distances in pixels were converted to that in physical units during one video frame period by calibrating with a stainless-steel ruler included in the recorded videos. In these images, 55 pixels corresponded to approximately 10 mm spacing on the ruler. With the travelling distances over one frame, the travelling speed for each of the six droplets was derived by using a physical equation - distance divided by time (**Table 1**).

Falling speed of a horizontally launched projectile

Approaching the end of the water spraying process, fewer water droplets came out of the bottle and their horizontal travelling speed gradually slowed, enabling us to capture the horizontally launched projectile motion of an individual

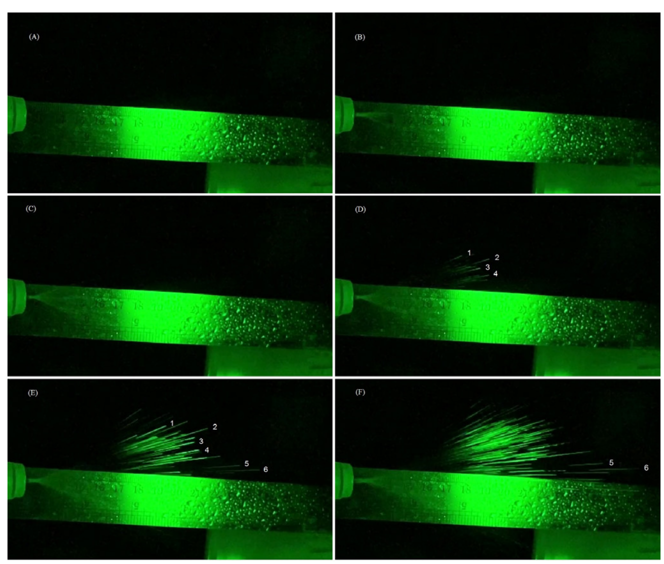

Figure 2. Six consecutive images taken when the water was initially sprayed out of the bottle. Six specific individual droplets can be clearly identified in the images from **(D)** to **(F)**, with droplet “1”, “2”, “3”, “4” marked in **(D)** and **(E)**, and droplet “5” and “6” marked in **(E)** and **(F).**

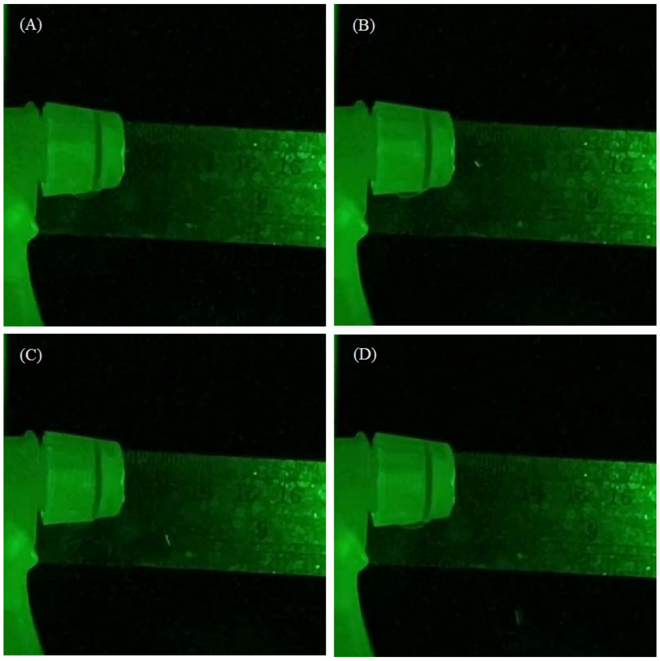

Figure 3. Four images – the 1st (A), 6th (B), 12th (C), and 18th (D) image – among 18 consecutive picture images that captures a water droplet experiencing a horizontally launched projectile motion.

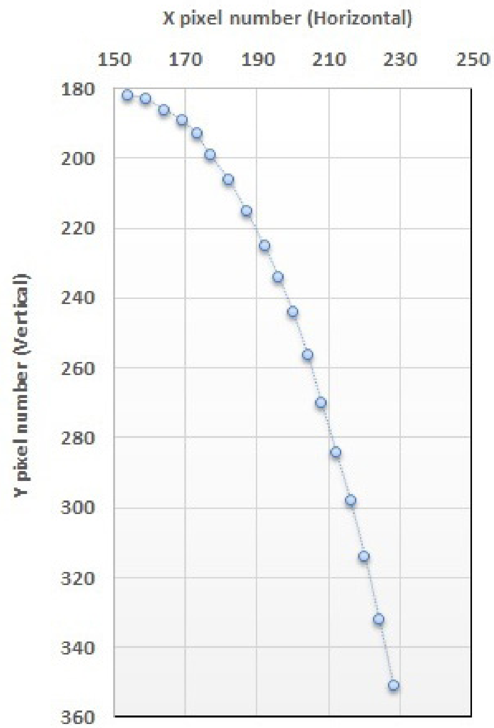

Figure 4. Horizontally launched projectile motion of the specific droplet presented in the pixel coordinate.

water droplet at a very slow speed. Four images – the 1st, 6th, 12th, and 18th image – among 18 consecutive images that capture a horizontally launched water droplet were chosen for analysis (labeled as **Figures 3A-3D**). In each of the video frames, a single pixel can be designated as the middle of the droplet during its trajectory, representing the location of the water droplet at that specific moment. Thus, the horizontally launched projectile motion of the single water droplet can be presented in pixel coordinates (**Figure 4**).

The pixel data confirmed that the water droplet was traveling in a horizontally launched projectile motion with a constant horizontal speed and a downward vertical acceleration. The horizontal speed was calculated to be 19.0 mm/s, which was averaged over 18 data points. The downward vertical acceleration was determined to be 985 mm/s2 using two data points – the vertical speed at one of the beginning frames (v_1=131 mm/s) and the vertical speed at the last frame (v_2=829 mm/s). Vertical acceleration was calculated through the equation $\boldsymbol{g = (v_2 - v_1)/\Delta t}$, where $\boldsymbol{\Delta t}$ is (17/240 s). Our calculated vertical acceleration is very close to the well-known value for acceleration of gravity, i.e., g = 980 mm/s^2, with a percent error of 0.4%.

Estimation of droplet landing and circumferential area

Assuming all water droplets were sprayed within an apex angle of 60 degrees at speeds similar to those derived above, we estimated how far and how wide they would be travelling in the air. We simplified the motion characteristics of water droplets to projectile motion under only the gravitational field of the earth without air resistance, which we believe to be valid as the droplet had a very small initial speed at the beginning moment of its free fall. The size of the circumferential area of the sprayed water droplets on the ground was estimated by calculating their projectile landing trajectories, with the assumption that all the droplets are spraying out within the same apex angle of 60 degrees and at different speeds. Given the fact that the droplets were expelled out from a height of 175 cm above the ground, we could estimate the size of droplet landing and circumferential area.

We visualized droplet trajectories as velocity vectors in a 3-dimentional coordinate system (**Figure 5**), which illustrates specific droplets travelling at a speed of v, an apex angle of θ, and at an azimuth angle of $\boldsymbol{\varphi}$**.** The azimuth angle is defined as the angle between the projecting velocity vector on the x-y plane and the y axis. From simple geometric calculations, we obtained three velocity vector components as follows,

$$v_y = v \cdot \sin\left(\frac{\theta}{2}\right) \cdot cos\varphi \qquad [1]$$

$$v_x = v \cdot \sin\left(\frac{\theta}{2}\right) \cdot sin\varphi \qquad [2]$$

$$v_z = v \cdot \cos\left(\frac{\theta}{2}\right) \qquad [3]$$

where v_y is the vertical component of the velocity vector in the y direction and v_x and v_z are horizontal components in the x direction and the z direction, respectively. Thus, the spatial location (X, Y, Z) of a droplet at a moment, t, and its falling time, T, from an initial height at Y_0 can be expressed as the following:

$$Y = Y_0 + (v_y - g \cdot t/2) \cdot t \quad [4]$$

$$X = v_x \cdot t \quad [5]$$

$$Z = v_z \cdot t \quad [6]$$

$$T = \frac{\sqrt{v_y{}^2 + 2gY_0 - v_y}}{g} \quad [7]$$

Using Eq. (1) through Eq. (7), we calculated the circumferential area of the water droplets landing on the ground with either a constant projectile speed (6.0 m/s) but different spray angles (i.e., apex angle of θ) (**Figure 6A**) or a constant spray angle (60 degrees) but different initial speeds (**Figure 6B**). The *x*-*z* coordinate is in the horizontal plane, with the *z*-axis being the center direction of the corn shape of water droplet spray and the *x*-axis being the direction perpendicular to the center direction of the corn-shaped spray. The calculation results indicate that the water droplets can be landed on the ground as far as ~8 m in the *z* direction, and as wide as ~6 m in the *x* direction.

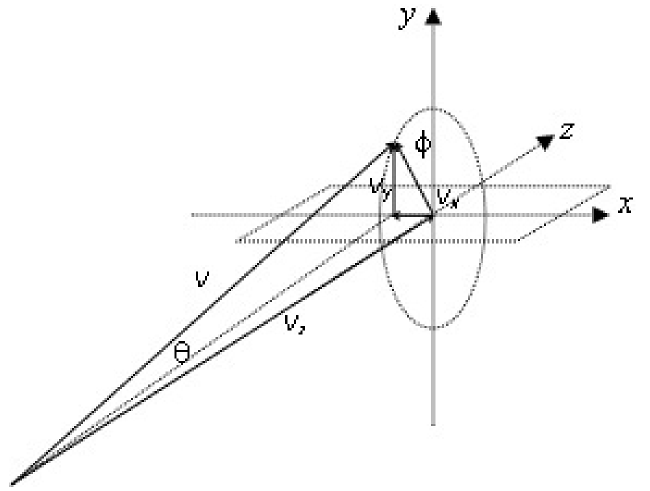

Figure 5. A velocity vector in a 3D coordinate system, illustrating a droplet travelling at a speed of v, at an apex angle of θ, and at an azimuth angle of φ.

DISCUSSION

In this study, we hypothesized that the iPhone X could be used as a low-cost high-speed photographic tool to accurately perform motion tracking and analysis of fast-moving objects. This hypothesis was demonstrated at home by using a low-power green laser, an iPhone X, and a water spray bottle to quantitatively investigate the projectile motion of droplets in the air.

A green laser beam was used as a light source for imaging because it has several benefits. Due to many decades of technological development, green lasers are the most powerful lasers operating in the visible spectral region, and they have become readily available on the market at a price much lower than other visible lasers at the same power level. The green laser wavelength is located within the most sensitive spectral region of image sensors used in smartphones or other consumer cameras. A green laser, just like all other lasers, has very high directivity - much higher than other light sources such as a flashing LED. This feature enables us to selectively illuminate only a small area of interest for imaging without ambient light interference that is detrimental to obtaining a high image contrast.

In this experiment, we demonstrated that a smartphone camera (iPhone X) can be used as a low-cost, high-speed, photographic tool to quantitatively investigate fast-moving objects. The projectile motion of water droplets was recorded by a smartphone with its Slow-Motion Video function. After converting the video clips into consecutive images at a rate of 240 Hz, motion tracking and analysis of the droplets were conducted in the pixel coordinate system, which were then converted to the physical unit coordinate by a calibration process. Travelling speeds and vertical acceleration of individual droplets were quantitatively obtained.

The calculated speeds of the six droplets (**Table 1**) ranged

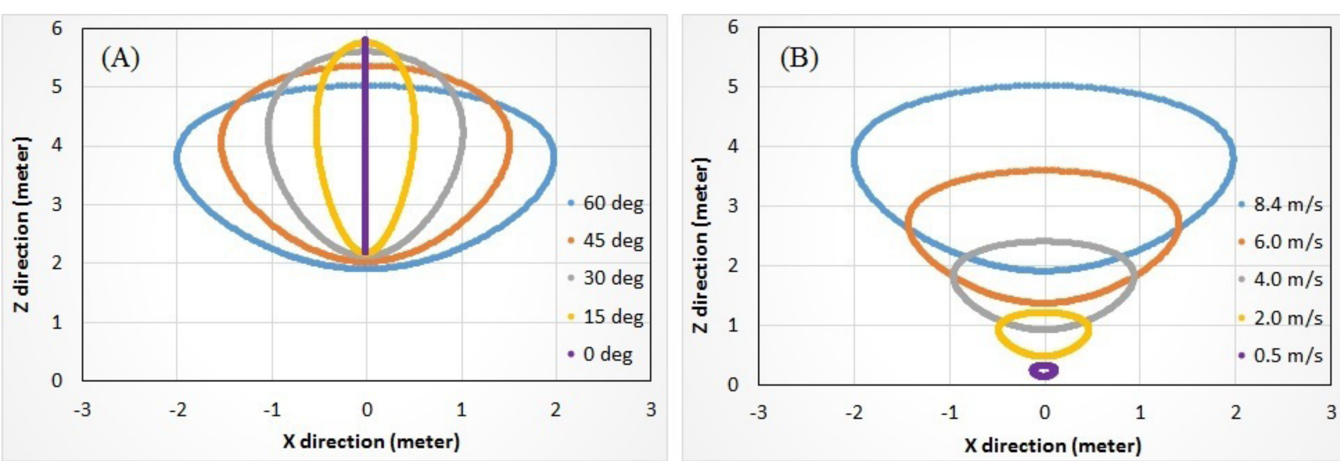

Figure 6. Calculated circumferential area of spray water droplets landing on the ground with (A) different apex angle of φ and (B) different initial speed *v*.

between 4 and 6 m/s, but these may not be their real speeds. These are likely just the velocity component of their speeds projecting to the plane that is perpendicular to the line of sight of the camera, since these two-dimensional images taken by the camera cannot detect the motion along the line of sight. If a droplet were to move out of the plane perpendicular to the line of sight, we could only measure the in-plane velocity component. Therefore, the differences between the measured speeds of the droplets does not necessarily mean these six droplets are moving at quite different speeds. It may mean their motions are in different directions but at quite similar speeds.

In addition to the uncertainty induced by out-of-plane motion, the calibration process could also bring an error in the speed measurements. We used the metric scale of the ruler (with a minimum deviation only at 1 mm) to calibrate moving distances of the droplets in the pixel coordinate system. Given the fact that the speed was derived from a distance measurement of a droplet travelling within one frame period, an uncertainty with only a half of the minimum ruler scale (i.e., 0.5 mm) in our calibration process could bring a systematic error of ±0.12 m/s on the speed measurements.

In the pixel coordinate system, one pixel corresponds to ~0.18 mm in the real-world coordinates since 10 mm on the ruler was equal to 55 pixels in the images. This could bring an uncertainty of ± 40 mm/s for speed measurements.

It is most likely that the timing jitter for video camera exposure could be neglected in comparison to the video frame rate and its exposure time for each frame. If this were the case, all our measurement errors should come from the measurement accuracy for moving distance in pixel coordinates and its conversion into physical unit coordinates. As mentioned above, error in the calibration process could bring systematic errors, which would be applied to all our measurements. One pixel in the pixel coordinate (corresponding to ~0.18 mm uncertainty for distance measurement in the real-world coordinates) could bring an uncertainty of ± 40 mm/s for speed measurements from the images taken at a rate of 240 fps. This level of uncertainty was significantly large as it is comparable to the speed of the droplet (**Figure 4**) measured at the beginning of falling, but not as significant when the speed increased by the end of falling.

Even with the slow-motion video function, a high-speed droplet was not captured in the images as a single dot, but instead displayed as an elongated droplet – a trajectory during the camera exposure time. Trajectory length of the moving water droplet shown in the video images should be proportional to its exposure time and the average speed over the exposure time. In **Figure 3D**, for example, the movement of the water droplet during the exposure time created a vertical displacement of 14 pixels in the *y*-axis of the pixel coordinate system with an uncertainty of ± 1 pixel, which is equivalent to 2.50 mm ± 0.18 mm. The average vertical speed in the last camera exposure from this trajectory can be estimated to be about 829 mm/s (**Figure 4**). Thus, we derived that the exposure time of the iPhone X under the Slow-Motion Video function is approximately 3 ms, which is about three quarters of the 4.16 ms period time of the video frame in the iPhone X camera, assuming a constant image capturing rate at 240 Hz.

It should be noted that all the measurements in this study were based on our assumption that the video frame rate was kept constant at 240 fps. If the video frame rate was varied or not as precise as its specification, accurate measurements would be difficult to ascertain. Given the fact that our calculated vertical acceleration of a horizontally launched droplet projectile was discrepant from the well-known acceleration of gravity, i.e., g = 980 mm/s^2, by a percent error of only 0.4%, however, it indicates that the video frame rate of the smartphone, to some extent, is precise.

In addition to the accuracy of the video frame rate, other factors could have impacted the uncertainty and error of our measurements in this study. These factors include the pixel resolution in the pixel coordinate system and the uncertainty of our calibration process. The measurement uncertainty caused by these two factors could be significantly improved by using an optical design with higher resolution, for instance, a better optical zoom for video recording. However, the fact that a two-dimensional image cannot sense the motion along the line of sight is a fundamental limit, which means that we can only measure the velocity components projecting onto the plane perpendicular to the line of sight, but not the velocity component along the line of sight.

Although there were the uncertainty and errors mentioned above, nonetheless, our experimental results in this study still suggest that a smartphone can be used as a low-cost, high-speed, photographic tool for quantitative investigation of fast-moving objects. With a better optical image design, those uncertainties could be significantly reduced, which would need more experimental demonstration.

MATERIALS AND METHODS

A low-cost green laser beam (<$100, <0.1 W power, OXLasers) was used as a light source for the experiment. For the sake of eye safety and further reduction of laser-induced ambient lighting, a low-power illuminating laser beam was delivered through a piece of a single-mode optical fiber with the laser source covered by a small cardboard box. The illuminating laser beam (<10 mW) was emitted directly from a cleaved end tip of the optical fiber, which was positioned in the horizontal direction. Without any other optics, the laser beam emitted from the optical fiber tip was shone in a ray with a divergent angle of about 8° in a dark room. Water was sprayed from a spray bottle in a perpendicular direction to the laser light path. As a reference for distance calibration in image coordinates, a ruler was placed in parallel to the direction of water spray just below the laser beam path, measuring the spread of the water droplets (**Figure 7**).

Water droplet spray processes were recorded with an iPhone X camera in the Slow-Motion Video operation mode.

After downloading these video clips onto a computer, an online software, Free Video to JPG Converter (9), was used to convert the Slow-Motion video clips into a group of consecutive images. From these images, we measured the spray angle of water droplets, identified locations of individual droplets and their trajectories in a pixel coordinate system by using Microsoft Paint. With the help of the ruler imaged together with the spray droplets, a calibration process could be performed, thereby converting the travelling distances in the pixel coordinate into that in the physical unit coordinate. Finally, we calculated the travelling speeds of the droplets with the travelling distances in the physical unit coordinate divided by travelling time.

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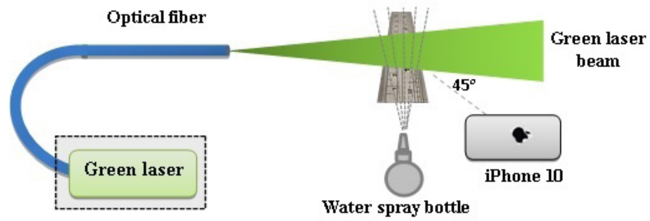

Figure 7. Experimental setup of capturing water droplet velocity and angle using a green laser and iPhone X.

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