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Article

Alterations of the [Fe/H] Values Modulate Light Curves by Absolute Magnitude in non-Blazhko RRab Lyraes

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SUMMARY

RR Lyraes are a type of variable star known for their short periods and their periodicity can be used as standard candles to determine the distances of interstellar objects they reside in. The correlation between the composition of RR Lyraes and their light curves is not well understood. This study investigated the relationship between the metallicity and resulting changes in the light curves of non-Blazhko RRab Lyraes. We analyzed 135 images of the 15 RR Lyraes throughout nine periods of the day by remotely accessing the Canary 2 telescope at the Slooh Canary Islands Observatory. We examined the images using multiple image analysis software programs to find locator stars, determine the apparent magnitudes of the RR Lyraes, and create light curves. We observed a negative exponential relationship between the [Fe/H] and the light curve amplitudes of the RR Lyraes, and a negative linear relationship between the [Fe/H] and the periods of the RR Lyraes. These results demonstrate that we can obtain a better idea of an RR Lyrae's composition based on solely its light curve and that characteristics of a light curve can be predicted based off of metallicity without time-consuming visual observations being required. These conclusions will assist in applying metallicity corrections to obtain more precise absolute magnitudes of the RR Lyraes so that they may be more effective standard candles.

INTRODUCTION

Variable stars are stars that change in luminosity over time (1). Approximately 200,000 of them have been discovered, and many of them have been placed into more than 20 different categories (2). Known to be old Population II stars, RR Lyraes are one of the most common types of variable stars, classified as intrinsic and pulsating (3). They are named after RR Lyrae, the brightest example and prototype star of the RR Lyrae variable star class. They are most prevalent in globular clusters, near the disk and halo of galaxies (4, 5). Located near the instability strip of the Hertzsprung–Russell diagram, the stars that demonstrate RR Lyrae variability are limited to those with specific physical properties (6). Their short pulsation periods make them the best candidates for variable star analysis, lasting from 0.1 to 2 days and ranging from 0.3 to 2 magnitudes in brightness (7-9). RR Lyraes are further divided into three subtypes based on the shape of their light curve – RRab (91% of all RR Lyraes), RRc (9%), and RRd (< 1%) (9, 10).

RR Lyraes are used as standard candles to calculate distances to interstellar objects (5, 7, 11, 12). The period-luminosity relation can calculate the luminosity of any variable star through information on its period (13, 14). The luminosity is expressed as $L = 4\pi d^2 m$, where L is the luminosity in solar units, d is the distance in parsecs, and m is the apparent magnitude (15).

Metallicity is defined as the fraction of a star's composition that is not hydrogen or helium – the crucial ingredients for stellar fusion (16). The iron-hydrogen ratio, or indicated by [Fe/H], measures metallicity by comparing the iron content of a star to its hydrogen content (16). This logarithmic unit is dependent on the Sun as shown by the following equation (where the asterisk represents the target star):

$$[Fe/H] = log\frac{(Fe/H)_*}{(Fe/H)_{Sun}}$$

This ratio can quantitatively measure the composition of a star and define if it is metal-rich or metal-poor. In the study by Nemec *et al.*, the [Fe/H] values were determined by analyzing the Fe I and Fe II in the spectra of the target RR Lyraes (19).

A significant property of RR Lyraes is the Blazhko effect, defined as a long-term, periodic fluctuation in period and amplitude of an RR Lyrae light curve that occurs in 30% of all RR Lyraes (**Figure 1**) (17). Without the Blazhko effect, RR Lyraes experience the same maximum and minimum flux values throughout multiple periods. The additional Blazhko effect periodicity produces noise in a data sample; thus, we

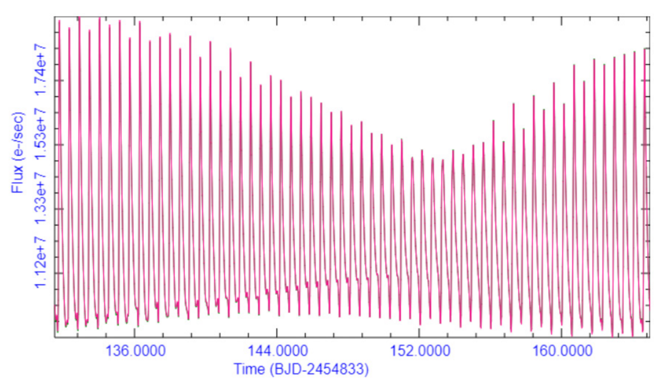

Figure 1: The RR Lyrae star demonstrating the Blazhko effect. The flux (directly correlated to luminosity) of the light curve's maxima and minima fluctuate over a long period of time (17). The figure is a screenshot from Kepler data (18).

RR Lyraes	Time (EST)								
	5:30 PM	6:30 PM	7:30 PM	8:30 PM	9:30 PM	10:30 PM	11:30 PM	12:30 AM	1:30 AM
NR Lyr	-2.4407	-2.6225	-2.7155	-2.6553	-2.8285	-2.7386	-3.0625	-4.3038	-2.7724
V782 Cyg	-0.5177	-1.0191	-1.0046	-0.8359	-0.7089	-0.4619	-1.1652	-0.3714	-0.2403
V784 Cyg	-0.1393	-0.1600	-0.6537	-0.7417	-0.5915	-0.3210	-0.1462	-0.1824	-0.4299
KIC 6100702	-4.1788	-4.3702	-4.5138	-4.6071	-4.2762	-4.2791	-4.2811	-3.9266	-3.7975
NQ Lyr	-2.4662	-3.1488	-3.0829	-3.5614	-3.2828	-3.1270	-3.2581	-3.9761	-3.8990
FN Lyr	-2.5064	-2.8878	-3.4605	-3.4975	-3.5534	-3.5439	-3.4425	-3.6199	-4.0223
KIC 7021124	-6.7220	-7.9242	-8.2350	-7.9687	-6.6598	-6.1535	-6.8909	-8.2090	-8.3501
KIC 7030715	-2.4142	-2.6743	-2.9465	-2.8180	-2.8512	-2.4074	-2.9916	-3.1295	-2.3096
V1510 Cyg	-1.9835	-2.6573	-2.4545	-2.3518	-3.2043	-3.1653	-2.6127	-2.5619	-3.5011
V350 Lyr	-8.0375	-8.7083	-8.4939	-8.4995	-8.4324	-8.1985	-8.1737	-8.0784	-8.2484
V894 Cyg	-5.1432	-4.8776	-4.8990	-4.7510	-4.7053	-4.7510	-4.7703	-4.9842	-5.0746
V2470 Cyg	-4.1512	-4.8740	-4.3359	-4.4442	-4.3696	-4.6481	-4.6973	-4.7677	-4.5483
V1107 Cyg	-0.2045	-2.5409	-2.0754	-1.2271	-1.2011	0.4527	-1.6672	-0.5812	-1.4576
V838 Cyg	-5.4219	-6.0141	-5.8607	-4.6875	-5.8335	-5.6845	-5.8842	-5.5810	-5.7519
AW Dra	-3.8583	-3.9051	-4.9322	-4.7302	-4.8139	-4.9149	-4.0427	-4.3134	-3.9249

Table 1: The absolute magnitudes of the 15 RR Lyraes found by running the Slooh images using AstroImageJ and using a locator star to compare magnitude.

investigated non-Blazhko RR Lyraes to refine our analysis and produce consistent results. We selected these non-Blazhko RRab Lyraes from a 2011 study by Nemec *et al*. (19).

The purpose of this analysis was to examine how the light curves of non-Blazhko RRab Lyraes would vary based on their metallicity. We hypothesized that the fluctuations of the absolute magnitude of the light curve will decrease as the iron content of the RR Lyrae increases. Based on the data, as the [Fe/H] values increased, the fluctuations of the absolute magnitude (as represented by the amplitude of the phase plot) of these variable stars decreased in a negative exponential relationship. Thus, the hypothesis was accepted. The rationale for this negative exponential trend may be because as the metallicity of a star increases, there is less room for hydrogen and helium as that room is taken up by metals. As that space is taken up, this results in less fusion so the RR Lyrae will have less energy for its light curve to fluctuate.

RESULTS

We investigated 15 non-Blazhko RRab Lyraes to analyze how [Fe/H] values of non-Blazhko RRab Lyrae variable stars affected their light curves. We took multiple images of each variable star via remote access and analyzed them using locator stars to calculate their apparent and absolute magnitude. We plotted the absolute magnitude data into light curves and compared by amplitude with other RR Lyraes in the study to examine the relationship between metallicity and the shape and amplitude of the graph. We hypothesized that the fluctuations of the absolute magnitude of the light curve will decrease as the iron content of the RR Lyrae increases because less fusion occurs with a decreased presence of hydrogen and helium, thus providing less energy for the light curve to produce a larger amplitude.

The analysis involved a total of 135 absolute magnitude values and 15 stars across nine different time slots from September to October 2018 (**Table 1**). These data points

RR Lyrae Name	[Fe/H]	Amplitude of Phase Plot
NR Lyr	-2.34	0.48
V782 Cyg	-0.47	0.35
V784 Cyg	-0.14	0.285
KIC 6100702	-0.35	0.325
NQ Lyr	-1.83	0.57
FN Lyr	-1.9	0.675
KIC 7021124	-2.08	0.88
KIC 7030715	-1.66	0.1725
V1510 Cyg	-1.83	0.3625
V350 Lyr	-1.84	0.3375
V894 Cyg	-1.79	0.15
V2470 Cyg	-0.71	0.14
V1107 Cyg	-1.56	0.645
V838 Cyg	-1.56	0.305
AW Dra	-1.74	0.485

Table 2: [Fe/H] and amplitudes of the phase plots of the 15 RR Lyraes found using VStar.

were plotted to produce light curves, and their amplitudes were calculated and compared to known [Fe/H] values by Nemec *et al.* (**Table 2**) (19). The amplitude of the light curves, or phase plots, represents the extent of the fluctuation of the RR Lyraes. The period of the RR Lyraes was determined by finding where the absolute magnitude returns to the same value as the first data point.

In **Figure 2**, the correlation between [Fe/H] and amplitude of the phase plots using a linear scale was a power law relationship with an equation of $y = 0.2268x^{-0.138}$, with an r^2 value of 0.1570 and $X^2 = 3.493$, $p = 0.9978$ with 14 degrees

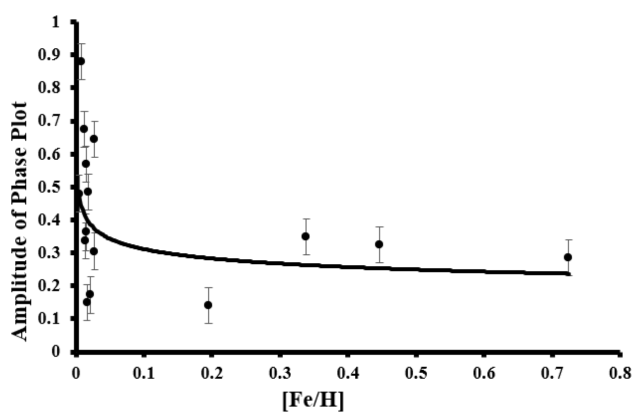

Figure 2: The relationship between [Fe/H] and the amplitude of the phase plots produced. Stars with a higher metallicity (less hydrogen and helium) showed less dramatic fluctuations in their light curves (smaller amplitude). Mean ± SD (n=15).

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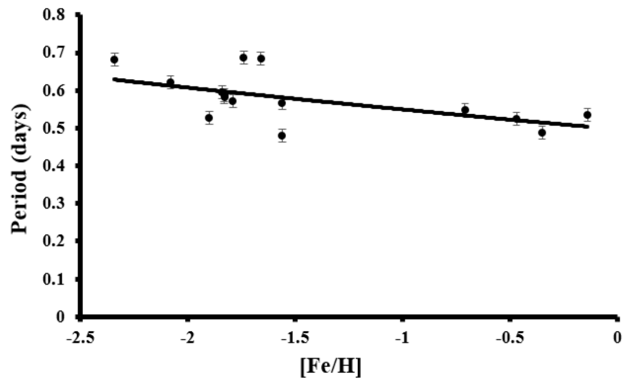

Figure 3: The relationship between [Fe/H] and the period of non-Blazhko RRab Lyraes. Stars with a higher metallicity were found to maintain shorter periods in their light curves. Mean ± SD (n=15).

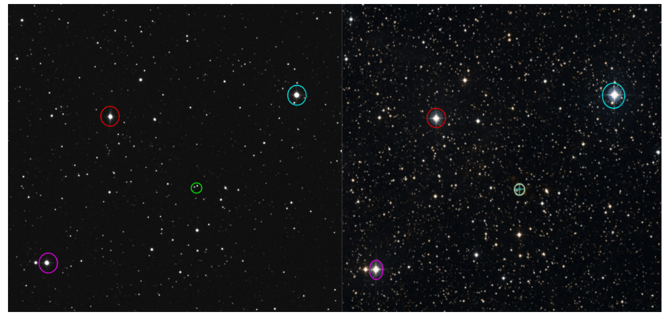

Figure 4: Comparison of Slooh image to SIMBAD image for identifying locator stars. To determine nearby locator stars of the RR Lyrae, we compared the image taken from Slooh and identified its location using the AladinLite View on SIMBAD. We then determined the coordinates and the apparent magnitude of locator star to find the apparent magnitude of the RR Lyrae.

of freedom. After removing outliers, the determined r^2 was 0.5584. When a logarithmic scale was applied, the line of best fit was $y = 0.2268e^{-0.318x}$, with an r^2 value of 0.2011 and $X^2 = 8.3835$, $p = 0.8684$ with 14 degrees of freedom. The conclusions of other studies are strongly connected to the correlation found in our study between [Fe/H] and absolute magnitude light curve amplitude. The concentration of data points on the left side of **Figure 2** was due to the RR Lyraes selected.

In **Figure 3**, the relationship between [Fe/H] and the period was a negative linear relationship modelled with the equation $y = -0.0582x + 0.494$ and an r^2 value of 0.3543. After removing outliers, the determined r^2 was 0.6013. The justification for this relationship is because RR Lyraes with fewer fusing elements are affected less by the natural process of atomic diffusion that can elongate a period in this type of variable star, as explained by Sandage *et al.* (20). This additional relationship proves how composition has a significant impact on an RR Lyrae's light curve. By being able to predict both the amplitude and the period of a RR Lyrae using just its [Fe/H] values, these relationships prove how metallicity can be a useful tool in predicting the light curves of RR Lyraes without extensive visual observation.

DISCUSSION

Through this study, the relationship between metallicity and the light curves of non-Blazhko RRab Lyraes was determined. We observed a negative exponential relationship between the [Fe/H] and the light curve amplitudes, and a negative linear relationship between the [Fe/H] and the light curve periods. These relationships prove how the composition of an RR Lyrae variable star has an impact on its light curve and thus can be utilized to provide better approximations of its light curve for use as a standard candle tool. The RR Lyraes used in this study were examined to be non-Blazhko RRab Lyraes in a 2011 study using the Kepler Space Telescope by Nemec *et al.* (19). Additionally, their [Fe/H] values were utilized in this analysis. The findings of Nemec *et al.* identified that non-Blazhko RRab Lyraes with lower [Fe/H] values tended to have longer periods (19). This relationship matched with a secondary analysis in this study, where the correlation between [Fe/H] and period was represented as a negative exponential trend. Although not exactly the [Fe/H] to amplitude relationship found in this study, Nemec *et al.* identified a negative linear relationship between [Fe/H] to [B–V] color index (19). As the [B–V] color index is closely related to absolute magnitude, this negative trend found by Nemec *et al.* likely is tied to the negative exponential trend found in the present analysis.

Multiple previous studies support the trend found in **Figure 2** regarding metallicity and RR Lyrae light curve amplitudes. In the findings of Fernley *et al.* in 1990, the [Fe/H] value of –1.52 corresponds to a visual absolute magnitude (MV) of 0.72 ± –0.10 (21). This point is approximately the midpoint of V1107 Cyg, one of the RR Lyraes in the present study, with a [Fe/H] of –1.56. Fernley *et al.* also identified a positive linear slope of 0.18 ± 0.03 when comparing [Fe/H] and visual absolute magnitude (21). Furthermore, a study by Demarque *et al.*

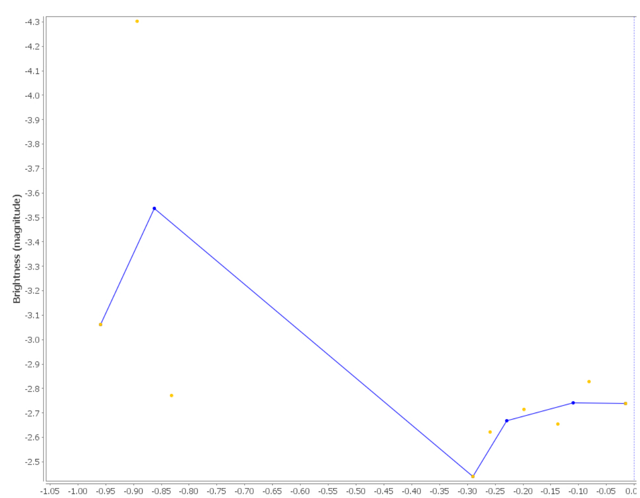

Figure 5: Resulting light curve of NR Lyr, one of the RR Lyraes analyzed. As the light curves produced by VStar display two periods (duplicates of the same data), only one period is shown here for clarity. The blue line represents the means of the data points from the analysis of the Slooh images. The x-axis is the phase while the y-axis is the brightness (in absolute magnitude).

looked into the use of horizontal branch models and the main sequence to further develop RR Lyraes as standard candles to globular clusters rather than using the period-luminosity relationship (22). By gathering data from the Hipparcos satellite, they concluded that the slope when depicting the relationship between visual absolute magnitude and [Fe/H] increased as [Fe/H] produced greater values. This provided evidence of a positive exponential relationship and supporting the study by Fernley *et al*. (21). These conclusions also verify the present study, since a larger absolute magnitude on a light curve's lower bound would produce smaller amplitudes with greater [Fe/H]. However, this draws discussions about a light curve's upper bound and how it would affect a phase plot's amplitude. A relationship identified by Fernley *et al*. was for Cepheids – the slope when comparing [Fe/H] and visual absolute magnitude was negative and linear at –2.15 ± 0.44, coming to show the variance amongst a wide spectrum of variable stars (21). A study on the specific Fourier components of RR Lyrae light curves by Norman R. Simon found that the particular Fourier phase parameter Φ_{21}, which shapes an RR Lyrae's light curve, decreases regularly with decreasing metallicity in short period stars but does not show this characteristic in long period stars (23).

In addition to the focus of our research in **Figure 2**, other papers support the additional relationship found in **Figure 3** regarding $10^{[Fe/H]}$ and period. In the findings by Sandage in 2004, a correlation across 140 stars between period *P*, metallicity, and amplitude range *AV* of RR Lyraes was found to be $[Fe/H] = -1.453\ AV - 7.990 \log P - 2.145$, a power law relationship written as a linear equation as the data were plotted with a logarithmic scale (24). This distinctly supports the current study as a negative relationship between [Fe/H] and both amplitude and period were observed in the 15 non-Blazhko RRab Lyraes analyzed.

In the future, increasing the sample size and diversifying the [Fe/H] values of the RR Lyraes would allow this correlation to be analyzed further. The large scatter in Fe abundance, which is typical in astronomical observations, certainly played a role in the determined r^2 value. Some additional errors may have resulted from the averaging procedure to calculate amplitude, the possible lack of data points to accurately determine period, and from varying weather conditions as the photos were taken across multiple days, which would have impacted the apparent magnitudes of the RR Lyraes in the images significantly.

One application of our conclusion would be to further utilize RR Lyraes as standard candles. Variable stars are often used to calculate distances because the period-luminosity relation facilitates the comparison between apparent magnitude and luminosity (13, 14). By analyzing the characteristics and light curves of RR Lyraes, scientists can better employ these variable stars as tools to find distances to galaxies, clusters, and other interstellar objects. Another application would be in research for stellar properties. Variable stars, especially RR Lyraes, contain quite extraordinary stellar properties that give them their characteristic light curve. In the future, further research on these variable stars such as analyzing the connection between composition and light curves for other variable star types or investigating the connection between composition and specific Fourier components of the light curve will provide a better comparison with normal stars and develop understanding of these unique stars.

MATERIALS AND METHODS

Obtaining Slooh Images

To gather data of 15 non-Blazhko RRab Lyraes, 135 images were taken via remote access by the 432 mm aperture Slooh Canary Two telescope located at the Institute of Astrophysics of the Canary Islands (IAC) in Tenerife, Canary Islands. Reservations were made for nine times during the day (eastern standard time): at 5:30 pm, 6:30 pm, 7:30 pm, 8:30 pm, 9:30 pm, 10:30 pm, 11:30 pm, 12:30 am, and 1:30 am. The 15 non-Blazhko RRab Lyraes include NR Lyr, V782 Cyg, V784 Cyg, KIC 6100702, NQ Lyr, FN Lyr, KIC 7021124, KIC 7030715, V1510 Cyg, V350 Lyr, V894 Cyg, V2470 Cyg, V1107 Cyg, V838 Cyg, and AW Dra. These variable stars were identified as non-Blazhko RRab Lyraes in a 2011 study by the Kepler Space Telescope by Nemec *et al*. (19).

Using AstroImageJ to Find Apparent Magnitude

By accessing the SIMBAD database, the AladinLite View provides information such as the apparent magnitude of the locator star (**Figure 4**). Using AstroImageJ (University of Louisville), the apparent magnitude of the variable star in the images can be found by comparing it to the given apparent magnitude of the locator star.

Parallax angles are offered on SIMBAD, which can help calculate the distance of the star in parsecs by using the equation:

$$d = 1/p$$

where *p* is the parallax angle in arcseconds and *d* is the distance in parsecs.

The apparent magnitude found in the collected images can be combined with the distance calculated to find the absolute magnitude via the distance modulus.

$$M = m + 5 - 5 \log d$$

where *M* is the absolute magnitude, *m* is the apparent magnitude, and *d* is the distance in parsecs.

Using VStar to Create and Analyze Light Curves

Data were put into a .txt file, with the Julian date in decimal format separated by a comma with the absolute magnitude, with each new line signifying a different time. Using VStar, a plotting program by the AAVSO, light curves (also known as phase plots) were created with means to display lines for the graph's shape (**Figure 5**). The light curve's amplitude was defined as the average of the absolute value of the difference between the maximum and minimum of the apparent magnitude values from **Table 1**.

Data Analysis

Data were plotted using Microsoft Excel. The line of best fit was calculated using the method of least squares which was also automatically calculated by Excel. The r^2 statistical test was implemented through Excel by using the RSQ formula on the dataset. The X^2 statistical test was also implemented through Excel by first producing a spreadsheet with the original data and the expected data (calculated using the line of best fit for the correlation) then using the CHISQ.TEST formula to find p (probability of independence). X^2 was calculated manually by using the formula (*observed value* - *expected value*)^2/(*expected value*) for each data point then adding up all the calculated results together. The degrees of freedom were calculated using (*rows* - 1) × (*columns* - 1). The correlating statistical tests were also calculated after removing outliers visually. The calculation for the error bars used the standard error method which is indicative of the data's standard deviation (n = 15 for **Figure 2** and **Figure 3**).

In **Figure 2**, the correlation between [Fe/H] and amplitude of the phase plots was determined by testing both a linear scale and a logarithmic scale. Since [Fe/H] is a logarithmic unit, the linear scale was determined by calculating $10^{[Fe/H]}$ for each RR Lyrae. The linear scale was implemented on the graph by placing $10^{[Fe/H]}$ on the x-axis and amplitude on the y-axis. After comparing different relationship types for the line of best fit on Excel, a power law relationship was found to visually be the best fit for the data. For a logarithmic scale, [Fe/H] was placed on the x-axis and amplitude on the y-axis.

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