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Determining the habitable zone around a star

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Summary

The goal of this project is to determine the habitable zone around a star, given the mass of the star. The determination of the habitable zone would apply only to stars in a singular solar system, as it examines the gravitational pull of one star. The research focuses on deriving the mass of the object being orbited by finding the velocity, acceleration, and the period of the orbit of the satellite, the object in orbit, using Newton's Law of Universal Gravitation, principles of circular motion, and Newton's Second Law of Motion. Then, utilizing software including Mathematica and LoggerPro and compiled research from areas of astronomy, biology, and physics, the boundaries for known life can be mathematically derived. The definition of life, used to set the parameters of the scope of the zone, depends on the ability for existing life to grow and reproduce. This project utilizes various sources of research and information to define the conditions in which life might exist, specifically the surface temperature of the satellite versus the distance from the star. This process, if combined with chemical and geographical information, could allow isolation of the most likely satellites for life in a singular solar system. Future work involving the study of chemical elements present in the satellite and its atmosphere may aid in further narrowing down the most likely habitable zones.

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Introduction

The search for planets that could harbor life has been a hot topic in the astronomical community today. For example, the National Aeronautics and Space Administration (NASA) utilized the Kepler Mission, or NASA Discovery mission #10, to survey regions of the galaxy for habitable zones (1). The Kepler Mission has developed the modern search for habitability by gathering statistics on the size, mass, orbital radii, and energy emission of various stars and their satellites. As of February 2012, the Kepler Science Team has discovered 1,790 host stars with 2,321 planet candidates (1). However, this interest in isolating habitable zones dates back to the late 1950s, when astrophysicists such as Su-Shu Huang and Hubertus Strughold dubbed the term "habitable zone", or its "ecosphere" (2). Various methods have been used in the past to calculate the habitable zone. Limits, such as the mean temperature appropriate for habitation by humans and the existence of water within a planet, have been established (2). However, many different aspects of science must be incorporated for a more accurate estimation of a habitable zone—from "stellar evolution, planetary dynamics, climatology, biology, and geophysics" to "planet formation processes and subsequent gravitational dynamics" (3).

The authors intend to investigate another method of defining a habitable zone by using a variable of mass to determine a distance from a central celestial body. The research focuses on combining Newton's Law of Universal Gravitation, which defines the amount of attraction that two bodies have with each other, with basic projectile characteristics like velocity, acceleration, and period. This approach is unique as it utilizes Newtonian physics as its core and supplements the derivation with biological and astronomical research. The question posed through this research is if a formula for defining a habitable zone of a singular celestial body can be mathematically derived using this novel method.

Results

For a star of mass M, the orbital radii of potential habitable satellites must fall within the following boundaries to sustain known life according to the outlined methods:

$$r_{\min} = \sqrt{\frac{QM^{3.5}}{(16\pi\sigma)(394.15K)^4}}$$

$$r_{\max} = \sqrt{\frac{QM^{3.5}}{(16\pi\sigma)(255.15K)^4}}$$

where Q = $\frac{L_{solar}}{M_{solar}^{3.5}}$

In order to verify the general accuracy of the results proposed, data from the Kepler Mission was compared to sample calculations with the equations proposed. Specifically, the Kepler-11 planetary system was studied. Kepler-11 is the first system discovered by NASA's Kepler Mission, which contained a star system with six orbiting planets. The equations derived above have been applied to Kepler-11.

Given that σ has a value of 5.67 × 10^{-8} $Wm^{-2}K^{-4}$, and that M stands for the solar mass, the habitable zone of the Kepler-11 system can be calculated as follows. Using data compiled from the NASA Kepler mission, the M for Kepler-11 is around 0.95 solar masses, with an error of 0.10 solar masses (14). The luminosity, L, is given as 1.13 solar luminosities with an error of 0.29 solar luminosities. One solar luminosity is equal to 3.839 × 10^{26} W. Substituting in the known values gives a r_{min} and r_{max} equal to, respectively, 7.472 ×10^{10} m and 1.896 × 10^{11} m. Comparing these distances to the planets in orbit in the Kepler-11 system and converting the standardized AU (Astronomical Unit) to m with a conversion factor from 149,597,871 km to 1 AU results in boundaries of distances between 0.499 AU and 1.267 AU. This radius indicates that none of the Kepler-11 extrasolar planets fall into the calculated range of habitability, as their distances range from 0.091 AU to 0.462 AU from Kepler-11 (14).

Discussion

The proposed hypothesis questioned whether an equation for a habitable zone could be developed from a foundation of Newtonian physics, a common topic in high school physics classrooms. This derivation required the integration of extensive research in other sciences (biology and astrophysics, for example); however, the derivation produced equations defining the maximum and minimum boundaries of a habitable zone. The real life application of this problem is apparent, given that organizations like NASA consider the pursuit of life in outer space an important and novel area of research. However, future work should include aspects of chemistry and geology. Because a star is not a perfect body, the equations derived for r_{min} and r_{max} can only serve as general equations to determine the zone around a star that yields a habitable temperature, which is crucial for the existence of life. However, mathematically, errors like the lack of a spherical body and the approximation of the shape of an orbit should also be considered. As astrophysics uses approximations for these equations, error is inevitably present.

In addition, as this habitability only sets parameters for life based on a certain level of energy received, the equation does not take into account the presence or lack of certain elements (*i.e.* liquid water or carbon), nor does it take into account the size nor substance composition of an atmosphere, which contributes to the amount of energy a blackbody takes in. The albedo, or reflectivity of the surface, would also contribute to this energy intake. Further research on how energy is affected by the presence of these elements would improve the equation and narrow the habitable zones in question. Additionally, it would be interesting to study the habitable zones of more bodies in the universe by examining the phenomena of binary or multi-solar systems, as this equation only takes into account the gravitational pull of a singular star.

Methods

Satellite Orbit

Studying certain aspects of a satellite can tell us the characteristics of the object that it is orbiting. In the proofs below, we isolate the mass of the object being orbited by deriving the velocity, acceleration, and the period of the orbit of the satellite.

Deriving the Velocity of the Satellite

The speed of a satellite in a circular orbit of radius r around an object of mass M_0 is:

$$v = \left(\frac{GM_0}{r} \right)^{0.5}$$

where G is the universal gravitational constant of 6.67 x 10^{-11} Nm^2kg^{-2} (4). For the purposes of this derivation, the orbit's shape is an approximation as generally planets follow an elliptical orbit.

This approximation is an application of Newton's Law of Universal Gravitation, which defines the force of any two objects of mass m_1 and m_2 to be of a force that is the magnitude of:

$$F = G\left(\frac{m_1 m_2}{r^2} \right)$$

where r is the distance between the masses. This Law of Universal Gravitation applies to all objects in the known universe, as "all objects in the universe attract all other objects in the universe by way of the gravitational interaction" (4). **Figure 1** provides an illustration of the Law of Universal Gravitation, where M_p is the mass of the orbited object, F_g is the force of gravity, v is velocity, a is acceleration, and y is the vertical difference between the centers of the orbited object and the satellite.

Using Newton's Second Law of Motion, we can then

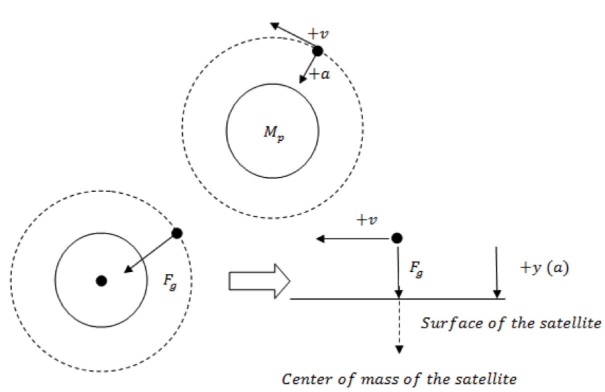

Figure 1. A force diagram illustrating the Newtonian physics concepts involved.

substitute in the following:

$$F_{net} = F_g = m_s a_g$$

where m_s is the mass of the satellite that is orbiting the larger object M_0 and a_g is the acceleration caused by gravity.

$$m_s a_g = G\left(\frac{m_s M_0}{r^2}\right)$$

Thus, the acceleration of the satellite can be derived:

$$a_g = G\left(\frac{m_s M_0}{r^2}\right)$$

Applying the principles of fundamental circular motion, the acceleration of an object in circular motion is equal to the velocity squared over the radius of the object's displacement:

$$a_g = G\left(\frac{M_0}{r_2}\right) = \frac{v^2}{r}$$

Using the circular motion equation to isolate the value of v from a_g leads to:

$$v = \sqrt{ar} = \sqrt{a_g r}$$

Therefore, substituting in Newton's Law of Universal Gravitation, the following equation can be derived:

$$\frac{v^2}{r} = G\left(\frac{M_0}{r^2}\right)$$

$$v^2 = G\left(\frac{M_0}{r}\right)$$

$$v = \sqrt{G\frac{M_0}{r}} = \left(\frac{GM_0}{r}\right)^{0.5}$$

To verify that this derivation is an appropriate value for the velocity, the units, the signs, and the sensibility of the value can be examined. Inserting the values of the units of each of the variables results in velocity having the unit of distance/time, which is appropriate for the units of velocity. The resulting sign should be positive, as the variables underneath the radical are all positive (G, M_0 and r inherently all are positive values), which is suitable for the value of the velocity. The resulting value is reasonable, as the net force method that Newton described was combined with principles of circular movement, which resembles orbital movement.

Deriving the Period of Orbit of the Satellite

The period of the orbit can be derived using similar methods. Using the derived velocity, and utilizing principles of circular motion, the period of the satellite can be described.

In circular motion, it is also known that:

$$v = \frac{2\pi r}{T}$$

where T is the period, and r is the radius of the orbit.

$$v = \frac{2\pi r}{T} = \left(\frac{GM_0}{r}\right)^{0.5}$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM_0}{r}}} = \frac{2\pi r^{1.5}}{\sqrt{GM_0}}$$

Checking the units of the calculated period of orbit, we can see that the unit simplifies to a unit of time, which is appropriate for measuring the period. The sign should be positive, as the signs within the radical and any constants would be positive. The sensibility stems from the utilization of the net force method from Newton's Second Law of Motion and also from principles of circular motion.

Using Mathematica, we can find the planets in our solar system and then accordingly find the orbit period, in seconds, for each of the planets in the solar system. These data, including the period and the orbital radius, can be put into LoggerPro and used to calculate the orbital speed, acceleration and the period using the equations described earlier. The relationships illustrated between the planets' mass and their orbital speed, acceleration and period reflect the equations derived

Fit	y-variable	Relationship	Coefficient	Units of Coefficient	Power of R
1	Acceleration, a	$a=\frac{GM_O}{r^2}$	GM_O	$\frac{m^3}{s^2}$	-2
2	Speed, v	$v=\sqrt{\frac{GM_O}{r}}$	$(GM_O)^{0.5}$	$\frac{m^{1.5}}{s}$	-0.5
3	Period, T	$T=\frac{2\pi}{\sqrt{GM_O}}r^{1.5}$	$\frac{2\pi}{\sqrt{GM_O}}$	$\frac{s}{m^{1.5}}$	1.5

Figure 2. A table outlining the relationships between acceleration, velocity, and period with the mass of the celestial body orbited

earlier. These data further verify the definition of the relationships between mass and acceleration, velocity, and period, as seen in **Figure 2**.

Using the exponent values of the radius, the equations of fit for each of the three graphs of speed, acceleration and period can be made. One of the methods by which the mass of the object was determined in this proof included using the relationships derived earlier for acceleration, velocity, and period to find the mass. Using these three relationships, the coefficient of the relationships from the value of the R to an exponent can be isolated, and the value of the coefficient as given by the fit curves derived by LoggerPro can be used. Using the relationships in the table, the mass of the object orbited, M_0, can be solved.

Using velocity as an example, after isolation of the coefficient, $(GM_0)^{0.5}$, mathematically the value of M_0 would be equal to the square of the coefficient divided by the value of the universal gravitational constant G.

Using data that applies to the Earth's solar system, verification that the mass of the object orbited would be equal to that of the sun can be made. These three relationships for the mass, derived as a function of the three parameters, are methods of expressing the same value. A method of deriving the mass using acceleration is outlined below.

Using the acceleration (4), we find that:

$$a_g = G\left(\frac{M_0}{r^2}\right)$$

$$M_0 = \frac{a_g r^2}{G}$$

Substituting in values for the orbit of Mercury, we find:

$$M_0 = \frac{\left(0.036\frac{m}{s^2}\right)\left(57909175000m^2\right)}{G}$$

= 1.810 x 10^{30} kg

Utilizing Mathematica, the *AstronomicalData* function of the software can be accessed to find the value of the mass of the Sun. The proximity of these calculations to the actual value shows the accuracy of the calculations from the motion of the orbit to derive the mass of the orbited object.

The following equation can be used to estimate the experimental error:

% Error = 100 · |Measured Value - Accepted Value| ÷ (Accepted Value). (5)

Thus, the experimental error for the calculations of the acceleration in finding the mass is:

% Error = 100 · |1.810 ×10^{30} kg – 1.988 ×10^{30} kg| ÷ (1.988 × 10^{30} kg)

= 8.953%

As this principle only applies to solar systems with solitary stars as the center of gravitational pull, this narrows its application to the one-third of stars that, like the sun, are single (6). The extension of this mathematical derivation of the mass of the orbited object becomes important when considering the existence of life on the satellites.

Satellites in single star systems derive their surface temperature from the effective temperature of the star they orbit. The distances from the stellar mass that they orbit set the magnitude of their surface temperature. If the maximum and minimum surface temperatures are within the broader set of temperatures at which known life exists, then we can hypothesize that life exists on the satellite.

Satellite Luminosity

The maximum known temperature at which life can exist, where life is defined as the continued ability to reproduce and grow, is approximately 121 °Celsius (8). In contrast, the coldest known limit at which life can exist is -18 °C (8). This definition of life is crucial to this research, as there are life forms that can exist in a stagnant state but not reproduce outside of this range (6).

The surface color of each of the stars reflects the stellar body's temperature (7). Also, "the temperature of a star, and therefore its color, actually depends on the amount of mass it has. Very massive stars, which can be over ten times the mass of the Sun, are the hottest and smaller stars, with less than half the mass of the Sun, are the coolest" (9). Therefore, using physics, the mass of the orbited object can be derived and the general effective temperature and color of the star can be found. After finding the effective temperature/color of the star, then the mean distance of the habitable zone can be determined. The habitable zone calculations can be checked using the equations derived earlier for velocity, acceleration, and period to verify the mass of the star it orbits.

Given that the mass of the orbited star has been derived, matching it with the known effective temperature for its size is the next step to find the mean distance of the habitable zone (**Figure 3**). Knowing these effective temperatures of the stars, the laws of physics can be used to determine how the mass, temperature, and distance of these celestial bodies are linked.

Luminosity and mass are directly proportional, and thus:

$$L \propto M^{3.5}$$

where L is luminosity (in units of solar luminosity, or 3.839 × 10^{26} W of energy), and M is the mass of the star in solar masses (1.989 ×10^{30} kg) (10). Using the values known for the sun, the equation can be solved for the proportionality. This rule would then apply to stars in general, the perfect star approximation.

Luminosity is proportional to 3.5 powers of the mass

THE SPECTRAL SEQUENCE			
Class	Spectrum	Color	Temperature
O	ionized and neutral helium, weakened hydrogen	bluish	31,500-49,000 K
B	neutral helium, stronger hydrogen	blue-white	10,000-31,500 K
A	strong hydrogen, ionized metals	white	7500-10,000 K
F	weaker hydrogen, ionized metals	yellowish white	6000-7500 K
G	still weaker hydrogen, ionized and neutral metals	yellowish	5300-6000 K
K	weak hydrogen, neutral metals	orange	3800-5300 K
M	little or no hydrogen, neutral metals, molecules	reddish	2100-3800 K
L	no hydrogen, metallic hydrides, alkalai metals	red-infrared	1200-2100 K
T	methane bands	infrared	under 1200 K

Figure 3. A table that outlines the temperature range in metric Kelvin of each class and color of star in the modern spectral sequence from http://people.highline.edu/.

(10). Therefore, one solar mass has one solar luminosity. Two solar masses have around 10 solar luminosities, 4 solar masses have around 100 luminosities, and 8 solar masses have around 1000 solar luminosities.

The Stefan-Boltzmann constant (represented by σ) links the effective temperature of the star to its mass. Given the relationship between the power emitted is proportional to the fourth power of the celestial body's effective temperature, the power emitted would equal $L \propto \sigma T^4$ (12). The constant σ has a value of 5.67 $\times 10^{-8}$ $Wm^{-2}K^{-4}$. Therefore, using the proportionality of luminosity and mass, and luminosity and temperature, the following equation is true:

$$M^{3.5} \propto \sigma T^4$$

Using data about stars of different spectral sequences, these equations can be verified. For example, a star from Class O, with a mass 50 times larger than the Sun would follow this proportion:

$$L \propto 50^{3.5}$$

And utilizing the estimation system of factors of ten, the following is true:

440413 solar luminosities $\propto$ 50 solar masses$^{3.5}$

This is a reasonable estimate, as 32 solar mass stars have 100,000 solar luminosities, and 64 solar mass stars have 1,000,000 solar luminosities (11).

Furthermore, from the Stefan-Boltzmann law that accompanies the constant, it is known that:

$$L \propto 4\pi R^2 \sigma T^4$$

because the luminosity is fixed per surface area of the star. This equation assumes that the star is spherical, and thus the surface area is $4\pi R^2$, where R is the star's radius.

Because of this proportionality,

$$L \propto 4\pi R^2 \sigma T^4 \propto M^{3.5}$$

which, ultimately, links the mass to the star's temperature T. It is reasonable that the mass is linked to the surface area of the star when observing temperature, because the surface area's luminosity (energy) would vary depending on how much energy the surface of the star could give off.

As defined previously, the known hospitable zone at which life can exist is from around -18 to 121 °C. However, since these universal calculations are derived in Kelvin, the boundaries of life must also be converted to Kelvin. Respectively, they are 255.15 and 394.15 Kelvin. These can be calculated using the ConvertTemperature function of Mathematica or by using the conversion relationship between Kelvin and Celsius: Kelvin = Celsius + 273.15.

In order to find the distance within which these temperatures occur for each amount of mass of the orbited star, we must then determine how the temperature of a blackbody planet depends on the size of its orbit, and then solve for the boundaries for a habitable zone. A blackbody planet is one that absorbs electromagnetic radiation (7). This model can be used since the planets that exhibit habitability must have the capacity to absorb energy to provide it to any living substances. This can be done using two calculations that have been described previously—the luminosity of the star and the radius of its orbit (13).

An extension of the Stefan-Boltzmann law leads to the following equation:

Temperature of the orbiting satellite (t)= $\left(\dfrac{L}{16\pi\sigma r^2}\right)^{0.25}$

where r is the orbital radius of the satellite around the star, σ is the Stefan-Boltzmann Constant, and L is the luminosity of the star. Of note, the temperature of the orbiting satellite t is not the same as the capital T used previously to describe the temperature of the star. Also, the orbital radius r is to be distinguished from the radius of the star R.

The following proportionality can now be applied:

$$t \propto L^{0.25}$$

to the former proportionality that was solved for to obtain this proportionality:

$$t \propto L^{0.25} \propto M^{0.875}$$

From this, we can conclude that the temperature of the satellite is thus proportional the mass of the orbited star.

As a real-world application, the ranges around each star in which known life could potentially exist can be solved. Life requires liquid water for survival. However, many types of life can exist at temperatures at which liquid water may not exist—from around -18 °C to 121 °C, as defined earlier. At these water is either frozen or evaporated, however, life continues to grow and reproduce, which is the definition of life for the purposes of this research project.

To properly express a temperature in Kelvin using a variable of a stellar mass in kilograms, the proportionality expressed before can be expressed in this manner:

$$\frac{L}{L_{solar}} = \frac{M^{3.5}}{M_{solar}^{3.5}}$$

$$L = \frac{L_{solar} M^{3.5}}{M_{solar}^{3.5}}$$

For convenience, a new variable, Q, will be defined for use in the following derivations:

$$Q = \frac{L_{solar}}{M_{solar}^{3.5}}$$

Therefore,

Temperature of the orbiting satellite (t)= $\left(\frac{QM^{3.5}}{16\pi\sigma r^2} \right)^{0.25}$

serves as an estimate of the temperature of the orbiting satellite in regards to the mass of the star, and the orbital radius. Since the mass of the star is generally in proportion to the temperature of the star, and thus the class of the star, to find the habitable zones of each class of star purely based on the mass of the star, the boundaries of the masses of each class of star must be used.

The temperature of the orbiting satellite must be between 255.15 Kelvin and 394.15 Kelvin, as described earlier.

t_{min} = 255.15 K = $\left(\frac{QM^{3.5}}{16\pi\sigma r^2} \right)^{0.25}$

t_{max} = 394.15 K = $\left(\frac{QM^{3.5}}{16\pi\sigma r^2} \right)^{0.25}$

The values of r can be solved in order to define the potentially habitable zone around stars of mass M.

$$t^4 = \frac{QM^{3.5}}{16\pi\sigma r^2}$$

$$16\pi\sigma t^4 = \frac{QM^{3.5}}{r^2}$$

$$r^2 = \frac{QM^{3.5}}{16\pi\sigma t^4}$$

$$r = \sqrt{\frac{QM^{3.5}}{16\pi\sigma t^4}}$$

Using the values 255.15 K and 394.15 K for t gives us functions that define the minimum and maximum values of r for there to be temperatures that can sustain life for a star of mass M.

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